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Disentangling risk aversion and intertemporal substitution through a reference level ☆

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Abstract

In the canonical CCAPM, the coefficient of relative risk aversion is constrained to be the inverse of the elasticity of intertemporal substitution. For theoretical and empirical reasons the two concepts should be disentangled. We suggest that disentangling may be obtained by replacing the future consumption stream not by a certainty equivalent of future utility, like in the recursive utility model of Epstein and Zin [1989. Econometrica 57, 937–969], but by an exogenous reference level of consumption, which, in a recursive way, assesses the expected future consumption.

JEL classification: G1; G11; G12; C1; C5

Keywords: Non-expected utility; Relative risk aversion; Elasticity of intertemporal substitution; Habit formation; Reference level; Resolution of uncertainty; Equity premium

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1. Introduction

In the standard consumption capital asset pricing model (CCAPM), a representative agent maximizes his time-separable expected utility. The curvature of a utility function captures two aspects of an agent's preferences. As the concavity of the function increases so does his aversion to risk as well as his desire to smooth consumption intertemporally. For a power utility function, it means that the coefficient of relative risk aversion is constrained to be the inverse of the elasticity of intertemporal substitution. This constraint is not supported by empirical observations since agents tend to exhibit an elasticity of intertemporal substitution which is less than the inverse of the relative risk aversion coefficient, as emphasized in Weil (1990). To disentangle the two concepts, Epstein and Zin (1989), often referred to as EZ hereafter, and Weil (1989) have proposed a recursive utility framework that generalizes the dynamic choice model under uncertainty of Kreps and Porteus (1978).

Epstein and Zin (1989) qualify this disentangling by stressing that the risk aversion parameter in their model should not be interpreted independently from the attitude towards intertemporal substitution.[1] However, after reading Epstein and Zin (1991) and a number of papers in the ensuing literature, one realizes that the estimates of the risk aversion parameter are often directly compared with the ones obtained in the standard CCAPM framework as in Hansen and Singleton (1983). We will argue that such a reading of the risk parameter in the Epstein–Zin model could lead to spurious interpretations of allegedly realistic low levels of estimated risk aversion. Moreover, Epstein and Zin's (1991) conclusion that "risk preferences do not differ statistically from the logarithmic specification" could be reinterpreted as an indication that the attitude towards intertemporal substitution does not differ statistically from the logarithmic specification. This reinterpretation is crucially important from an economic point of view since it allows to distinguish myopia in consumption-saving decisions from myopia in portfolio allocation (Giovannini and Weil, 1989).

Following Garcia et al. (2002), henceforth GRS (2002), we suggest that the requested disentangling may alternatively be obtained by replacing the future consumption stream not by a certainty equivalent of future utility but by an exogenous reference level of consumption, which, in a recursive way, assesses the expected future consumption. Therefore, risk aversion is now defined with respect to the unpredictable discrepancy between actual consumption and this reference level (a quantity independent of the attitude towards risk) and not with respect to the forthcoming level of recursive utility which still mixes attitudes towards risk and intertemporal substitution.[2] However, this disentangling maintains the same unambiguous definition of the elasticity of intertemporal substitution for deterministic future streams of consumption.

In this new framework, preferences are therefore represented by a generalized von Neumann–Morgenstern utility specification whereby satisfaction is derived from consumption relative to an external reference level as well as from this reference level itself. This specification is closely related to several concepts in the literature. In external habit formation models, utility is measured

[1] Weil (1989, 1990) is more assertive about the distinct nature of the two preference parameters. The risk parameter is clearly interpreted as the Arrow–Pratt coefficient of relative risk aversion for timeless gambles, while another unrelated constant parameter is defined as the elasticity of intertemporal substitution for deterministic consumption paths.

[2] Kőszegi and Rabin (2004) also propose a model of reference-dependent preferences. In their model, a person's utility is the sum of an intrinsic consumption utility that is independent of the reference level (the classical one) and of a gain-loss utility, where gains and losses in utility are measured with respect to the consumption utility from the reference level.

with respect to consumption relative to a time-varying habit or subsistence level either in ratios (Abel, 1990, 1996) or in differences (Constantinides, 1990; Sundaresan, 1989; Campbell and Cochrane, 1999), among others. Several variables have also been added to the utility function besides consumption: leisure (Eichenbaum et al., 1988), public expenditures (Aschauer, 1985), durable goods (Startz, 1989), wealth (Bakshi and Chen, 1996; Smith, 2001). Recently, Carroll et al. (2000), in a growth and saving model, proposed a specification in which the agent can derive utility both from the level of consumption relative to a reference level and from the absolute value of this reference level.

To recover a stochastic discount factor (SDF) which is observationally equivalent to the Kreps and Porteus specification in the recursive utility framework of Epstein and Zin (1989), we establish a structural link between this reference level of consumption and the return on the market portfolio. However, we emphasize that, although observationally equivalent, the two models deliver different measures of risk aversion. As in Barberis et al. (2001), the introduction of a reference level actually changes the measure of risk and in turn the level of risk aversion needed to explain the observed risk premium. In GRS (2002), we generalize the reference level and make it depend on past consumption as well as on the return on the market portfolio. Therefore, we embed both habit persistence and the recursive utility Kreps–Porteus model in the same SDF.

Section 2 presents the issue of disentangling risk aversion from the elasticity of intertemporal substitution in a recursive utility framework. In Section 3, we introduce a von Neumann–Morgenstern expected utility framework that separates differently the two concepts. In particular, the risk aversion parameter is not the same as the measure of risk aversion in the recursive utility specification. The latter is shown to depend on the intertemporal elasticity of substitution, while the former does not. Section 4 concludes.

2. Disentangling risk aversion and intertemporal substitution via recursive utility

Our focus of interest is the modeling of a preference ordering between stochastic consumption processes $C = (C_t)_{t \geqslant 0}$. Following Duffie and Epstein (1992a), it is quite natural to consider that a utility function U is *risk averse* if, for all processes C in some domain:

$$U[C] \leqslant U[EC],$$

where EC denotes the deterministic process defined by $[E(C)]_t = E[C_t]$. A more difficult issue is to assess the level of risk aversion of a given utility function U in this intertemporal context. A measure of this level is crucial for contributing to the empirical debate surrounding asset pricing puzzles. For instance, the equity premium puzzle amounts to consider that the level of risk aversion needed to reproduce the observed risk premium on equity is not reasonable. One step in the direction of quantifying risk aversion has been performed by Duffie and Epstein (1992a) through the notion of comparative risk aversion. They define this concept as follows.

Definition 1. A utility function U^* is said to be *more risk averse* than U if it rejects any gamble that is rejected by U, that is for any stochastic process C and any deterministic process $\bar{C}$ in some domain: $U[C] \leqslant U[\bar{C}] \Rightarrow U^*[C] \leqslant U^*[\bar{C}]$.

In other words, if U prefers a deterministic sequence $\bar{C}_t$, $t \geqslant 0$, to a stochastic consumption process C_t, $t \geqslant 0$, a fortiori U^* will prefer the deterministic path. As acknowledged by Duffie and Epstein (1992a), this definition is not innocuous. To be comparable according to this definition, U^* and U must rank deterministic programs identically.

In particular, one cannot give a sense to the statement "U^* is more risk averse than U" if U^* and U feature different temporal preferences, either for immediate versus late consumption (subjective discounting) or for consumption smoothing (elasticity of intertemporal substitution). This is indeed a fundamental impossibility result about disentangling risk aversion and intertemporal substitution. The only way to escape this general impossibility is to be more specific about the utility model. Epstein and Zin (1989), Weil (1989), and Duffie and Epstein (1992a, 1992b) put forward the recursive utility framework in discrete time and continuous time, respectively.

First introduced by Koopmans (1960) in a deterministic setting, the recursive relation:

$$V_t = W[C_t, V_{t+1}] \tag{2.1}$$

specifies the utility index V_t at time t as a function of the consumption C_t in period t and the utility index V_{t+1} of future consumption. The function W has been called an aggregator by Lucas and Stokey (1984). It defines both the rate of time preference and the elasticity of intertemporal substitution. For instance, the time-additive separable (TAS) utility function[3] corresponds to the aggregator:

$$W[C, V] = u(C) + \beta V, \tag{2.2}$$

where β is the subjective discount factor. In the isoelastic case, $u(C) = \frac{C^\rho - 1}{\rho}$, $\rho \leqslant 1$, $\rho = 1 - \frac{1}{\sigma}$, $\sigma > 0$ is the elasticity of intertemporal substitution. The issue of interest is to extend Eq. (2.1) to uncertain consumption streams. Then, the future utility index V_{t+1} appears itself random at time t (we will denote it $\tilde{V}_{t+1}$ to stress that it is stochastic) and cannot be plugged into (2.1) without a preliminary treatment.

In other words, we must look for a generalization of (2.1) which admits the latter equation as a particular case when the future random value of $\tilde{V}_{t+1}$ is known at time t. The solution proposed by Epstein and Zin (1989) appears to be quite natural in this respect. They consider that the agent first computes the certainty equivalent $m(\tilde{V}_{t+1} \mid I_t)$ of the conditional distribution $(\tilde{V}_{t+1} \mid I_t)$ of $\tilde{V}_{t+1}$ given the information at time t and then combines the latter with C_t via the aggregator W:

$$V_t = W\big[C_t, m(\tilde{V}_{t+1} \mid I_t)\big]. \tag{2.3}$$

They refer to Kreps and Porteus (1978) to study (2.3) under the assumption that m is an expected-utility based certainty equivalent such as

$$m(\tilde{V}_{t+1} \mid I_t) = f^{-1}\big[E\big[f(\tilde{V}_{t+1}) \mid I_t\big]\big], \tag{2.4}$$

where they call f a von Neumann–Morgenstern utility index.

This terminology is motivated by the fact that the utility functions defined by (2.3) and (2.4) conform with expected utility theory when ranking timeless gambles. To see this, let us consider a lottery on a sequence (C_{t+h}), $h \geqslant 0$, of current and future consumption that is genuinely timeless because the two following conditions are fulfilled. First, randomness is about just one particular future consumption (C_{t+H}) for given H, while all the other ones are known at time t. For notational simplicity, we assume that for any $h \neq H$, $C_{t+h} = C^*$, a given level of consumption. Second, the uncertainty at time t about C_{t+H} has no temporal features. Basically, the value of C_{t+H} appears to be random at time t but is going to be known no later than time $(t+1)$.

Then, with the aggregator (2.2), the utility index V_t at time t is given by

$$V_t = u(C^*) + \beta m\big[(1-\beta)^{-1} u(C^*) + \beta^{H-1}\big\{u(C_{t+H}) - u(C^*)\big\}\big]. \tag{2.5}$$

[3] See Becker and Boyd III (1997) for a review of aggregators and their properties.

We deduce from (2.5) that it is true that m characterizes the risk aversion preferences for timeless gambles. Typically, for a given level of risk involved in future consumption C_{t+H}, different people will value more or less such a gamble depending upon their level of risk aversion included in m or, equivalently, in the von Neumann–Morgenstern utility index f.

This risk aversion assessment appears at first sight to be fairly well disentangled from the other features of preferences since the rate of time preference and the elasticity of intertemporal substitution, as described respectively by β and the function u, do not play an important role in this argument. Of course, the risk exposure is not assessed directly in terms of consumption units C_{t+H}, but only through its concave transformation $u(C_{t+H})$. Yet, no genuinely perverse effect results form this concave scaling.

However, if one thinks about more general temporal gambles, it is no longer true that, as commonly believed, m and f will determine the degree of risk taking in portfolio choice problems. We argue that the disentangling of risk aversion and intertemporal substitution is not fully done in the recursive utility framework (2.3) and (2.4). To explain this intuitively, we will rely on the analysis of Alvarez and Jermann (2004), who establish a clear distinction between the related concepts of equity premium and cost of consumption uncertainty. The marginal cost of consumption uncertainty is defined, as we did above, from the time t return until maturity of an asset with a single risky payment C_{t+H} at $(t+H)$. However, the consumption equity premium (for an equity with dividends equal to consumption) is defined from the time t shadow price of an asset which pays the full stochastic process of dividends $[C] = [C_{t+h},\ h \geqslant 0]$. In order to control for preferences for the timing of uncertainty resolution, let us maintain the assumption that all uncertainty about this process is revealed at time $(t+1)$. Then the relevant utility index is

$$V_t = u(C_t) + \beta m\left[\sum_{h=1}^{\infty} \beta^{h-1} u(C_{t+h})\right]. \tag{2.6}$$

Assume for expositional simplicity that the stochastic process $[\beta^h C_{t+h},\ h > 0]$ is stationary and ergodic. Then, the higher the elasticity of intertemporal substitution featured by the function u is, the more the individual is able to consider the stochastic process $[\beta^h C_{t+h},\ h > 0]$ as almost equivalent to its smoothed counterpart $[C^*_{t+h},\ h > 0]$ defined by

$$C^*_{t+h} = \lim_{H=\infty}\left(\frac{1}{H}\right)\sum_{j=1}^{H} \beta^j C_{t+j}. \tag{2.7}$$

But, by the law of large numbers, the smoothed consumption process is no longer risky. In other words, a high elasticity of intertemporal substitution allows one to think in terms of intertemporal diversification and substantially lowers the level of risk that is significantly borne in a formula like (2.6). A more formal argument could be easily developed thanks to a boundedness assumption on the rescaled consumption process $[\beta^h C^{\rho}_{t+h} \leqslant l]$ as put forward in Epstein and Zin (1989).

This remark is of course highly relevant when it comes to solving the equity premium puzzle since it implies that m does not provide a meaningful assessment of the individual risk aversion. In other words, one cannot claim to have successfully solved the puzzle when a reasonable level of risk aversion (as described by m or f) is obtained in a representative agent model consistent with (2.3) and (2.4). It may only mean that risk aversion has been underestimated through its m (or f) characterization since the agent, with a sufficiently high elasticity of intertemporal substitution, might have perceived that the risk was not so high because of temporal diversification.

This possibility of temporal diversification explains that, as acknowledged by Duffie and Epstein (1992a, 1992b), the significance of the function m for comparative risk aversion arises only for a given elasticity of intertemporal substitution. Our argument goes further though. It would be illusory to rely on a plausible estimate of the risk aversion coefficient in an Epstein–Zin model of asset prices to consider that the equity premium puzzle has been solved. It all depends on the value of the elasticity of intertemporal substitution. The higher it is, the more spurious the inference will be. Following Alvarez and Jermann (2004), this line of reasoning would assimilate the equity premium with the cost of consumption uncertainty even though they are clearly distinct, both conceptually and quantitatively. They argue that the steepness of the term structure and the persistence of the shocks are two of the features that make the equity premium different from the marginal cost of consumption uncertainty.

We will propose in the next section an expected utility functional form which explicitly takes into account the degree of persistence of the shocks. For this reason, it will provide a clear disentangling between risk aversion and the elasticity of intertemporal substitution. The resulting asset pricing model will be observationally equivalent to the one of Epstein and Zin (1989), but it will modify the definition of the risk aversion measurement. This different definition will avoid the aforementioned shortcoming of recursive utility, that is the underestimation through m of the true level of risk aversion when the elasticity of intertemporal substitution is high.

This expected utility model has another advantage: it only refers to an individual who is neutral with respect to the timing of uncertainty resolution. Indeed, in addition to the temporal aspects of preferences, captured by β and the function u, and the risk aversion measure given by m, a third aspect of preferences should concern the timing of resolution of uncertainty. Actually, if in the above example we assume now that the risky consumption flow C_{t+H}, $H \geqslant 2$, is going to be revealed only at time $(t+2)$, we realize that the utility index at time t is different from (2.5). In other words, the definition of (β, u, m) characterizes the conditions under which early or late resolution is preferred. Thus, as recognized by Epstein and Zin (1989), this “latter aspect of preferences seems intertwined with both substitutability and risk aversion.” While they suspect that this “reflects the inherent inseparability of these three aspects of preference rather than a deficiency” of the framework, the model proposed in the next section will give more support to the requirement of disentangling preferences for the timing of uncertainty resolution from substitutability and risk aversion. In contrast with Epstein and Zin (1989), this aspect of preferences does no longer seem implied by the comparison of disentangled levels of elasticity of intertemporal substitution and risk aversion. Therefore, one can envision a more general model which would not only disentangle risk aversion from intertemporal substitution, but also describe independently the timing of uncertainty resolution.

3. A consumption CAPM with a reference level

In GRS (2002), we develop an intertemporal expected utility model where the representative agent derives utility from consumption measured relatively to a reference level and from this reference level itself:

$$V_t = \left[\lambda(1-a)\right]^{-1} \sum_{h=0}^{\infty} \delta^h E_t \left\{ \left[\frac{C_{t+h}}{S_{t+h}}\right]^{1-a} [S_{t+h}]^{\lambda} \right\}, \tag{3.1}$$

where the reference level S_t is considered as external to the agent and E_t denotes a conditional expectation given the information at time t. Depending on the specification of the reference level and on the constraints imposed on the various preference parameters, we show in GRS (2002) that this model produces several SDFs that have been used in the empirical asset pricing literature. We will see below how it should be modeled to obtain a SDF which is observationally equivalent to the one derived by Epstein and Zin (1989).

Our argument rests essentially on the fact that the reference level provides a way to extend intertemporal choice of consumption without uncertainty to risky consumption streams. When no uncertainty prevails, the future sequence of the reference level at time t, S_{t+h}, $h \geqslant 0$, coincides with the optimal future consumption values:

$$S_{t+h} = C_{t+h} \quad \text{identically for } h \geqslant 0. \tag{3.2}$$

In a risky environment, we just generalize condition (3.2) in terms of conditional expectations[4]:

$$E_t[S_{t+h}] = E_t[C_{t+h}] \quad \text{for all } h \geqslant 0. \tag{3.3}$$

Therefore, we can interpret S_{t+h} as the reference level the agent has in mind at time t to decide his risk-taking behavior. In the spirit of Abel (1990) and Campbell and Cochrane (1999) models of external habit formation, some macroeconomic variables which belong to the agent's information set at time $(t+h)$ may affect the assessment of the reference level S_{t+h}. In the model of Barberis et al. (2001), when the representative agent's consumption C_{t+h} coincides in equilibrium with the $\bar{C}_{t+h}$ aggregate per capita consumption at time $(t+h)$ (viewed as exogenous to the investor), the reference level of consumption will integrate the gain or loss the agent experiences on his financial investments between $(t+h-1)$ and $(t+h)$. In all these examples, the growth rate $\frac{S_{t+h}}{S_{t+h-1}}$ of benchmark consumption between dates $(t+h-1)$ and $(t+h)$ may include some information contemporaneous with C_{t+h}.[5]

Since the reference level is viewed as external by the agent, the resulting Euler conditions lead to a generalized CCAPM with the following SDF:

$$M_{t+1} = \delta \left[\frac{C_{t+1}}{C_t}\right]^{-a} \left[\frac{S_{t+1}}{S_t}\right]^{a+\lambda-1}. \tag{3.4}$$

Given condition (3.2), the parameter λ in (3.1) can unambiguously be interpreted in terms of intertemporal elasticity of substitution, with $\lambda = 1 - \frac{1}{\xi}$, where ξ denotes the agent's elasticity of intertemporal substitution. Such a SDF implies that the definition of the reference level must produce conditional expectations that are not only constrained by (3.3), but also consistent with the observed asset prices.

Let us consider first the market portfolio pricing condition. If we denote by $R_{M,t+1}$ the return on the market portfolio observed at time $(t+1)$, we get:

$$E_t\left\{\delta \left[\frac{C_{t+1}}{C_t}\right]^{-a} \left[\frac{S_{t+1}}{S_t}\right]^{a-\frac{1}{\xi}} R_{M,t+1}\right\} = 1. \tag{3.5}$$

[4] Kószegi and Rabin (2004) also characterize their reference level not in terms of the status quo, as it is usually done, but in terms of expectations.

[5] Campbell and Cochrane (1999) also specify that the consumption habit moves in response to current aggregate consumption and not, as in many habit formation models, in proportion to the last period consumption. Since habit is considered as external, the reference level S_{t+h} may even be defined as a function of C_{t+h}.

Condition (3.5) shows that covariation between the reference level and the market return may compensate for the lack of covariation between consumption and the market return. This extension of the traditional consumption-based asset pricing model may help to solve several asset pricing puzzles associated with aggregate data. As stressed by Barberis et al. (2001), such an extension has some behavioral foundations since it captures the idea that the degree of loss aversion of the investor depends on his prior investment performance. To make even more explicit this tight relationship between the reference level and investment performance as measured by the market return, we will refer to a log-linearization of conditional moment restrictions (3.3) and (3.5) (see Epstein and Zin (1991) and Campbell (1993) for similar interpretations based on a log-linearization of the Euler equations). Conditional expectations are computed as if the vector:

$$(\Delta c_{t+1}, \Delta s_{t+1}, r_{M,t+1}) = \left(\ln\left[\frac{C_{t+1}}{C_t}\right], \ln\left[\frac{S_{t+1}}{S_t}\right], \ln R_{M,t+1}\right) \tag{3.6}$$

were jointly normal and homoskedastic given the information available at time t. Conditions (3.3) and (3.5) at horizon 1 become:

$$\begin{aligned} &E_t[\Delta c_{t+1}] - E_t[\Delta s_{t+1}] = \kappa_1, \\ &-aE_t[\Delta c_{t+1}] + \left(a - \frac{1}{\xi}\right)E_t[\Delta s_{t+1}] + E_t[r_{M,t+1}] = \kappa_2 \end{aligned} \tag{3.7}$$

for some constants κ_1 and κ_2. Equivalently, these two restrictions say that both $[\Delta s_{t+1} - \Delta c_{t+1}]$ and $[\Delta s_{t+1} - \xi r_{M,t+1}]$ must be unpredictable at time t. We will now see that the Epstein and Zin (1989) pricing model is observationally equivalent to the particular case of the CCAPM with a reference level where $[\Delta s_{t+1} - \xi r_{M,t+1}]$ is not only unpredictable but constant:

$$\Delta s_{t+1} = \xi r_{M,t+1} + \kappa \tag{3.8}$$

for some constant κ. In other words, the benchmark growth rate of consumption is log-linearly determined by the current value of the market return, with a slope parameter equal to the elasticity of intertemporal substitution.[6] It should be noted that all results will remain valid for the general asset pricing model defined by (3.5) and (3.8), without any reference to log-normality.

Given the specification (3.8) of the reference level, it is clear that the parameters δ and κ cannot be separately identified from the SDF in (3.5) only. We will therefore reparameterize it in the following way:

$$M_{t+1} = \delta^* \left[\frac{C_{t+1}}{C_t}\right]^{-a} [R_{M,t+1}]^{a\xi - 1}, \tag{3.9}$$

where $\delta^* = \delta e^{\kappa(a - \frac{1}{\xi})}$.

At first sight, we obtain a SDF which is observationally equivalent to the one derived by Epstein and Zin (1989) with the TAS aggregator and isoelastic functions u and f. Yet there are several important differences in the interpretation of the two SDFs.

Let us start with the market return which enters both SDF specifications. In the GRS model, it appears because the investor links the benchmark consumption to the market return. In the recursive utility framework, it appears because the investor cares about the timing of uncertainty

[6] Note that this is in accordance with the portfolio separation property generally implied by homotheticity of preferences (see Epstein and Zin, 1989), whereby optimal consumption is determined in a second stage, after the portfolio choice has been made.

resolution. Actually, in (3.1), the utility index is defined in terms of conditional expectations of future random variables given the information available at time t, and therefore, the investor appears to be neutral with respect to the timing of uncertainty resolution. In this respect, our approach is closer to Bakshi and Chen (1996), who put forward the hypothesis that investors accumulate wealth not only for the sake of consumption but also for wealth-induced social status. Typically, if the reference level S_t were equal to aggregate wealth, a non-zero difference between λ and $(1-a)$ would lead to Model 1 of Bakshi and Chen (1996) where absolute wealth is status. This explains why Bakshi and Chen (1996) also put forward a kind of observational equivalence between their model and Epstein and Zin (1989). However, our approach does not reduce to theirs because they implicitly consider that the rate of growth of aggregate wealth coincides with the market return, which is not true in general. They differ because the share of wealth invested is not constant.[7] On the contrary, Lettau and Ludvigson (2001) have emphasized the prominent role played by the consumption-wealth ratio as a state variable to summarize the relevant conditioning information.

The GRS model may be better understood by reference to the external habit formation literature. The agent derives utility both from the level of consumption relative to the state variable S_t and from the absolute value of this reference level, which is similar to a habit. In the spirit of the external habit formation literature, the coefficient a is then interpreted as the risk aversion coefficient. This interpretation immediately raises the following question. Since the SDF (3.9) is observationally equivalent to the one of Epstein and Zin (1989) with isoelastic functions u and f ($u(C) = (\frac{1}{\rho})(C^\rho - 1)$ and $f(V) = (\frac{1}{\alpha})(V^\alpha - 1)$), it should shed some light on the difficult issue of risk aversion assessment in the context of the recursive utility model of Epstein and Zin (1989). Actually, the exponent of $\frac{C_{t+1}}{C_t}$ in this model (see Epstein and Zin, 1989, Eq. (6.6), p. 958) is[8]:

$$\alpha\frac{(\rho-1)}{\rho} = \frac{\alpha}{1-\sigma}, \quad \text{since } \sigma = (1-\rho)^{-1}. \tag{3.10}$$

By identification of the exponent of the growth rate of consumption in (3.9) and (3.10), we deduce that the quantity $(1-\alpha)$, instead of being interpreted as a risk aversion measure, should be seen as

$$(1-\alpha) = a + 1 - a\sigma. \tag{3.11}$$

Several comments are in order. First, the ability of the recursive utility model to disentangle risk aversion and intertemporal substitution is questionable. Actually, it is only in the standard expected utility model case, when σ is the inverse of the relative risk aversion parameter a, that $(1-\alpha)$ can be interpreted as the RRA coefficient. Even more problematic is the fact that $(1-\alpha)$ becomes negative whenever σ is greater than $\frac{1}{a}+1$. Note that this lack of disentangling manifests itself even without resorting to our interpretation of a as a risk aversion parameter. The natural

[7] Bakshi and Chen (1996) can be interpreted as a particular case of the GRS model with a unit elasticity of intertemporal substitution. Smith (2001) proposes to extend Bakshi and Chen (1996) by taking into account both the concern about wealth-induced status and the attitude towards the timing of uncertainty resolution. However, it is a simple i.i.d. economy in which the stochastic variation in the invested share of wealth cannot be accommodated.

[8] It should be noted that the IES is identical in both SDFs and is therefore denoted σ for both cases.

requirement of a negative exponent for $\frac{C_{t+1}}{C_t}$ in the SDF implies that the alleged risk aversion parameter $(1-\alpha)$ and $\frac{1}{\sigma}$ should be on the same side of 1.[9]

Second, as soon as σ is greater than $\frac{1}{a}$, the risk aversion measure $(1-\alpha)$ underestimates the genuine risk aversion parameter a. Hence, a relatively high level of elasticity of intertemporal substitution may spuriously indicate a moderate risk aversion. If, as documented by Mehra and Prescott (1985), the model can replicate the equity risk premium only for a high level of risk aversion, say $a = 20$, even a moderate elasticity of substitution, say 0.8, will dramatically lower the perceived risk aversion in the recursive utility model: $(1-\alpha) = 5$.

Of course, expressing concerns about the recursive utility model does not imply that the alternative model we propose is valid. For tests of its empirical validity, we refer the reader to GRS (2002).[10] However, it is important to stress that, apart from the issues related to the interpretation of risk aversion and the attitude towards the timing of uncertainty resolution, the two models are mutually consistent. Indeed, taking (3.11) into account, we can rewrite the exponent of the market return in the Epstein–Zin SDF as

$$\left(\frac{\alpha}{\rho}\right) - 1 = a\sigma - 1. \tag{3.12}$$

By identification with the exponent of the market return in (3.9), we see that our SDF is nothing but a reparametrization of the Epstein–Zin's one with: $\xi = \sigma$ (and $\lambda = \rho$). This coincidence between the parameters of intertemporal substitution of the two models is fully consistent with the interpretation sketched above. It also sheds some interesting light on the issue of myopic portfolio choice. Giovannini and Weil (1989) have stressed that a unit elasticity of intertemporal substitution implies a form of (rational) myopia in consumption and savings decisions but not in portfolio allocation. Actually, a unit elasticity of intertemporal substitution ($\lambda = 0$) reduces our general SDF (3.4) to

$$M_{t+1} = \delta\left[\frac{C_{t+1}}{C_t}\right]^{-a}\left[\frac{S_{t+1}}{S_t}\right]^{a-1} \tag{3.13}$$

and, given the reference level specification (3.8)[11]:

$$M_{t+1} = \delta\left[\frac{C_{t+1}}{C_t}\right]^{-a}(R_{Mt+1})^{a-1}. \tag{3.14}$$

This formula can be seen as the explicit solution of Eq. (B.5) in Giovannini and Weil (1989) for the particular case of conditional log-normality. Except for logarithmic risk preferences ($a = 0$ for us and $\alpha = 0$ for them), the Euler equations for portfolio choice with $\sigma = 1$ correspond in general to neither the static CAPM nor the CCAPM. Giovannini and Weil (1989) develop their argument rigorously but it is hard to do with the EZ SDF. One gets the spurious feeling that the exponent of consumption growth in the SDF $\left(\frac{\alpha}{1-\sigma}\right)$ is well defined and non-zero

[9] This is not the case in Bansal and Yaron (2004). To emphasize the role of economic uncertainty in asset pricing, they set σ equal to 1.5 and $(1-\alpha)$ equal to 10.

[10] We estimate in particular a model where the reference level growth rate is determined both by past consumption growth rates (as in habit formation models) and by the return of the market portfolio (as in the Kreps–Porteus specification of the recursive utility model of Epstein and Zin, 1989). The parameters of this specification are economically plausible and estimated with precision.

[11] With or without this log-linearization, this result is fully consistent with the point made by Giovannini and Weil (1989).

in the limit cases only if one maintains the equivalence: $\sigma = 1 \Leftrightarrow \alpha = 0$. This is the equivalence between myopic consumption-saving decisions and logarithmic risk preferences. This issue is relevant empirically. For instance, Epstein and Zin (1991) conclude that risk preferences do not differ statistically from the logarithmic specification but, by the same token, they find estimates for the elasticity of intertemporal substitution that are not statistically different from 1 for the two first sets of instruments (the ones which, according to the authors, give the most sensible results). Another interesting case is the portfolio applications in Campbell and Viceira (2002). To be able to disentangle myopia in consumption-saving decisions from myopia in portfolio allocation, they set $\sigma = 1$ while allowing any value for the risk aversion parameter. While they are certainly right to do so for economic interpretations, this is strictly speaking not consistent with the EZ parameterization, as it can be seen not only through our interpretation but also through Appendix B of Giovannini and Weil (1989). By contrast, the exponent a in our SDF (3.13) and (3.14) is in no way restricted by the condition $\sigma = 1$.

To summarize, the only difference between the GRS approach and the Epstein and Zin (1989) recursive utility model concerns the incorporation of the preferences for intertemporal choice without uncertainty in a risky environment with constant relative risk aversion. While the recursive utility approach replaces the future random utility index by its certainty equivalent, we believe it is preferable to replace upstream the future consumption flows by an external benchmark produced by the first-order conditions for optimal consumption. This benchmark determines the role of the time preference parameters while the risk aversion parameter matters only insofar as uncertainty prevents the agent from meeting his benchmark.

Of course, our benchmark specification could be questioned when log-linearization biases and volatility predictability are no longer negligible. While extensions can be envisioned in this regard, our new insight on risk aversion assessment in the recursive utility model is useful for addressing asset pricing puzzles. In terms of risk premium for individual assets, log-linearization of the pricing equations resulting from our SDF gives

$$E_t[r_{i,t+1}] - r_{f,t+1} = a\sigma_{ic} - (a\sigma - 1)\sigma_{im}, \tag{3.15}$$

where σ_{ic} and σ_{im} denote the covariances of asset i returns with consumption growth and market returns, respectively. This asset pricing model is observationally equivalent to the one of Epstein and Zin (1989) but the interpretation of the coefficients and the orders of magnitude deemed to be reasonable differ. The coefficient of σ_{ic} should be interpreted as a risk aversion parameter, which means, in particular, that it is constrained to be non-negative. Following the recursive utility parametrization, this would not always be the case since $a = (1 - \sigma)^{-1}(a^* - 1)$, where $a^* = (1 - \alpha)$ is the risk aversion measure. In addition, the coefficient of σ_{im} is $(a\sigma - 1) = (1 - \sigma)^{-1} \times (a^*\sigma - 1)$, which can take very large values for seemingly realistic values of the coefficient a^*.

4. Conclusion

In this paper, we have proposed a generalized expected utility framework which disentangles risk aversion and intertemporal substitution in an alternative way to the recursive utility framework proposed by Epstein and Zin (1989). Although observationally equivalent, the two models may lead to significantly different conclusions regarding the well-documented asset pricing puzzles. In particular, a plausible value of the elasticity of intertemporal substitution smaller than one, but not too close to zero, can conceal, within a recursive utility framework, a very large implied value for risk aversion.

One of the advantages of our specification is its flexibility. In GRS (2002), we show that it can reproduce several SDFs that have been proposed in the empirical asset pricing literature. In particular, it covers all external habit formation approaches and can be seen as a generalization of Campbell and Cochrane (1999). Disappointment or loss aversion can also be accommodated in our framework given the right specification of the reference level (see GRS, 2002). Moreover, it allows to specify new SDFs that can potentially better explain asset prices. In GRS (2002), we propose a new SDF based on habit persistence and the return on the market portfolio which appears to be supported by the data.

This generalized expected utility framework basically maintains the assumption of investor neutrality with regard to the timing of uncertainty resolution. Yet, as emphasized by Kreps and Porteus (1979), temporal preference for consumption is only an induced preference. Is earlier resolution of uncertainty better simply because it permits an adaptive choice of the individual activities or do individual preferences for consumption streams include a genuine subjective preference for earlier or later uncertainty resolution? A more general equilibrium model could then justify embedding our von Neumann–Morgenstern utility with respect to a reference level into a recursive framework with a clear formulation of the timing of outcomes of lotteries and resulting actions taken by the agent. Such a model might provide an answer to the question raised by Epstein and Zin (1989): is there some inherent inseparability of the three aspects of preferences: risk aversion, intertemporal substitution and concern for the timing of uncertainty resolution? This paper might have provided a first step in finding an answer by a better disentangling of risk aversion from intertemporal substitution without any implication about preference for earlier or later resolution of uncertainty.

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