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RISK AVERSION, INTERTEMPORAL SUBSTITUTION, AND THE TERM STRUCTURE OF INTEREST RATES

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SUMMARY
We build and estimate an equilibrium model of the term structure of interest rates based on a recursive utility specification. We contrast it with an arbitrage-free model, where prices of risk are estimated freely without preference constraints. In both models, nominal bond yields are affine functions of macroeconomic state variables. The equilibrium model accounts for the tent-shaped pattern and magnitude of coefficients from predictive regressions of excess bond returns on forward rates and the hump-shaped pattern in the term structure of volatilities, while the reduced-form no-arbitrage model does not account for these important features of the yield curve. Copyright © 2011 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Most recent models of the term structure of interest rates are formulated in an arbitrage-free framework, whereby bond yields are affine functions of a number of observed and unobserved state variables that capture the sources of uncertainty in the economy. When only three latent factors are specified, the traditional interpretation following Litterman and Scheinkman (1991) links them to the level, slope, and curvature of the yield curve.[1] To understand the channels through which the economy influences the term structure, Ang and Piazzesi (2003) and Ang *et al.* (2006) add measures of inflation and real activity to the latent factors. The joint dynamics of these macro factors and the latent factors are captured by vector autoregression (VAR) models. In these models based only on the absence of arbitrage, risk premiums for the various sources of uncertainty are obtained by specifying time-varying prices of risk that transform the risk factor volatilities into premiums. The prices of risk, however, are estimated directly from the data without accounting for the fact that investors' preferences and technology should impose some constraints between these prices.

Our first contribution is to build a flexible equilibrium term structure model that links the preferences of the representative agent to the risk premiums of the risk factors that affect bonds. In terms of preferences, our model has two key features. First, the subjective discount parameter is time-varying.[2] We follow Obstfeld (1990) and link the subjective discount parameter to the short-term rate of interest, which is a state variable in our framework. A second key feature is a recursive utility structure, where the coefficient of relative risk aversion (CRRA) is disentangled

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[1] Dai and Singleton (2003) and Piazzesi (2009) provide thorough surveys of this class of models.
[2] Preferences with time-varying rates of time preference were introduced by Uzawa (1968) and have been extended by Epstein (1987). In these specifications, the marginal utility of consumption in a given period can vary with consumption in other periods.

from the elasticity of intertemporal substitution (EIS). We show that these two ingredients are necessary to explain bond risk premiums.[3]

A second contribution of the paper is to estimate the preference parameters and other crucial parameters for risk premiums. For their arbitrage-free model, Ang *et al*. (2006) propose a sequential estimation strategy. They estimate the VAR parameters first and then they minimize the least-square distance between the observed market yields and the model-implied ones in order to recover the prices of risk ($K^2 + K$ parameters for a model with K state variables). A similar strategy with our equilibrium model would allow us to recover only the two free preference parameters characterizing the recursive utility structure. This is obviously not enough to price well bonds at various maturities and to capture their return dynamics. Our formulas for the equilibrium bond risk premiums indicate that the AR coefficients of the first-order VAR describing the dynamics of the state variables (K^2 parameters) are key. Therefore, we use the observed market yields to infer these parameters, consistent with investors' perception of the state variable dynamics. The estimated preference parameters are economically plausible: in the non-expected utility model, the CRRA is around 6 and the EIS is around 0.36. It also produces bond yields and returns that match empirical stylized facts.

Statistical tests of the expectations hypothesis conclude that bond risk premiums vary with the shape of the yield curve and that excess bond returns are indeed predictable. In particular, Cochrane and Piazzesi (2005) run predictive regressions of 1-year excess returns on forward rates and find that the forecasts are highly significant. They find a robust tent-shaped pattern of slope coefficients for all maturities, with regression R^2 around 35%. A major finding of our paper is that the unrestricted version of our recursive utility equilibrium model can account for these violations of the expectations hypothesis, but not the restricted expected-utility model nor the reduced-form no-arbitrage model. The non-expected utility model produces mean slope coefficients with the correct pattern across maturities and the actual coefficients found in the market data are also well covered by the respective confidence intervals. Further, the hump-shaped pattern in the term structure of unconditional volatilities of yields and yield changes is apparent in the recursive utility model but not in the other two models. These findings support the hypothesis that equilibrium restrictions on risk factor prices are essential to limit the variability of the stochastic discount factor (SDF) in order to reproduce the dynamics of bond risk premiums. To further highlight this point, we modify the reduced-form model by replacing the AR coefficients of the estimated VAR in the latter by those estimated directly from the bond yields in the equilibrium model. This modified reduced-form model is now competitive with the recursive utility model in reproducing empirical stylized facts for bond returns, but it does not provide any improvement over the equilibrium approach despite the fact that we still estimate 30 risk prices to fit bond yields. This suggests that the information contained in bond yields about the dynamics of the state variables is central for reproducing stylized facts. In the simple reduced-form model, the numerous prices of risk are not able to adjust the estimated VAR-implied coefficients to reproduce the stylized facts.

State-dependent preferences are commonly used in asset pricing.[4] Our model is a special case of the model by Melino and Yang (2003), who generalize the standard recursive utility framework by allowing the representative agent to display state-dependent preferences and show that such preferences can account for moments on equity and the risk-free rate. To explain the term structure of interest rates, we allow for a time-varying subjective discount parameter, but keep CRRA and EIS time-invariant. The fact that we link the subjective discount parameter to the short-term rate

[3] Gregory and Voss (1991) show that recursive preferences alone do not offer a solution to the bond premium puzzle put forward by Backus *et al*. (1989), which is that the representative agent model with power utility can account for the average risk premiums in holding-period bond returns and forward rates only with implausibly large values of the CRRA.
[4] For bond pricing models, see Piazzesi (2009) and the references therein.

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of interest as in Obstfeld (1990) is motivated by the central role played by the short rate in the determination of bond prices. Indeed, most models in the bond pricing literature find a way to introduce the short-term rate in the SDF.[5]

The recent paper by Piazzesi and Schneider (2006) is certainly the closest to ours. They also derive equilibrium yield curves under recursive preferences, but their approach differs in several respects. First, they express the SDF in terms of news about future consumption instead of a proxy return for the market portfolio as we do.[6] Then, they specify an exogenous state-space system for consumption growth and inflation and set values for the preference parameters in order to infer the equilibrium yields. Here we estimate the parameters of a VAR system including consumption growth and inflation and the preference parameters that rationalize the observed yields. In this sense, we follow more closely the no-arbitrage approach (e.g. Ang *et al.*, 2006), where prices of risk that rationalize the observed yields are extracted.

Wachter (2006) also proposes a consumption-based model of the term structure of interest rates, where nominal bonds depend on past consumption growth through habit, and on expected inflation. Her model is essentially the same as the habit model of Campbell and Cochrane (1999), but the sensitivity function of the surplus consumption to innovations in consumption is chosen so as to make the risk-free rate a linear function of the deviations of the surplus consumption from its mean. Moreover, Wachter calibrates her model so as to make the nominal risk-free rate in the model equal to the yield on a 3-month bond at the mean value of surplus consumption.

The rest of this paper is organized as follows. Section 2 describes the equilibrium model with recursive utility preferences that is used to price bonds. We also specify the dynamics of the macroeconomic variables that influence the yields. Section 3 is dedicated to model estimation and evaluation. We specify the benchmark no-arbitrage model, the data sources, and the econometric methodology used to estimate the parameters and ultimately to compute the yields. Section 4 presents the empirical implications for the term structure of volatilities and the analysis of risk premiums. Section 5 offers some concluding remarks.

2. EQUILIBRIUM MODEL

The recursive utility model of Epstein and Zin (1989) allows for a constant Arrow-Pratt CRRA that can differ from the reciprocal of the EIS. Melino and Yang (2003) generalize the standard recursive utility framework by allowing the representative agent to display state-dependent preferences and show that such preferences can account for moments on equity and the risk-free rate. To explain the term structure of interest rates, we only allow for a variable rate of time preference; CRRA and EIS remain time-invariant and can thus be deemed structural in our framework. As in the standard framework, we consider an infinitely lived representative agent who receives utility from the consumption of a single good. In any period t, current consumption, C_t, is deterministic but future consumption is uncertain. The agent's utility is characterized by the recursive relation:

$$U_t = \left(C_t^{\rho} + \beta_t (E_t[U_{t+1}^{\alpha}])^{\frac{\rho}{\alpha}} \right)^{\frac{1}{\rho}} \tag{1}$$

[5] Examples include the popular bond pricing models of Vasicek (1977) and Cox *et al.* (1985), and the larger class of affine term structure models (Duffie and Kan, 1996; Dai and Singleton, 2000), in which the SDF is a function of multiple factors, in addition to the short rate.

[6] An equity index proxy better captures the empirical links between equity and bond markets and reduces the value of the estimated risk aversion parameter since equity returns are more volatile than news about future consumption (Piazzesi and Schneider need a value of 43 to match the average 1- and 5-year yields). Such a proxy has been used in all GMM-based tests of the recursive utility model.

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where the expectation is conditional on time-t information and $0 < \beta_t$ is a subjective discount (or time preference) parameter that depends on an exogenous state variable. Melino and Yang show that, as in the standard recursive utility case, $\alpha \leq 1$ can be interpreted as a relative risk aversion parameter, with the degree of risk aversion increasing as α falls ($1-\alpha$ is the CRRA) and the parameter ρ can be interpreted as reflecting substitution, since $1/(1-\rho)$ is the EIS.

The agent's objective is to choose feasible consumption and portfolio shares to maximize utility. Melino and Yang show that maximizing U_t subject to a wealth accumulation constraint yields the following SDF used by the agent to discount future payoffs and determine current asset prices:

$$m_{t+1} = \beta_t^{\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{\gamma(\rho-1)} (R_{t+1})^{\gamma-1} \tag{2}$$

where R_{t+1} is the one-period gross rate of return on the market portfolio and $\gamma = \alpha/\rho$. Equation (2) shows that the SDF is a geometric weighted average of the rate of growth of consumption and the rate of return on the market portfolio—the agent's optimal portfolio; see Appendix A of Melino and Yang (2003) for the derivation. Market prices can then be expressed by the usual expected-value relation:

$$p_{it} = E_t[m_{t+1} g_{i,t+1}] \tag{3}$$

where p_{it} is the price of any asset i and $g_{i,t+1}$ is its future payoff. Note that the quantity in (2) is a strictly positive random variable that must satisfy (3). As in Epstein and Zin (1991), we use the returns on an equity index as a proxy for those of the aggregate wealth portfolio.

The basic asset pricing equation can also be written as $1 = E_t[m_{t+1} r_{t+1}]$, where $r_{t+1} = g_{t+1}/p_t$ defines gross returns. Gross returns can be defined either in nominal or real terms; correspondingly, the SDF must then be expressed in nominal or real terms. In nominal terms, the SDF in (2) becomes

$$m_{t+1}^{\$} = \beta_t^{\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{\gamma(\rho-1)} (R_{t+1})^{\gamma-1} \left(\frac{P_{t+1}}{P_t} \right)^{-1} \tag{4}$$

where P_{t+1}/P_t is the gross rate of inflation between periods t and $t+1$; P_t is the nominal price index at time t. Let $r_t = \log R_t$ represent the logarithm of the return on the proxy for the aggregate wealth portfolio, $\pi_t = \log P_t/P_{t-1}$ the rate of inflation, and $c_t = \log C_t/C_{t-1}$ the rate of consumption growth.

We close the specification of the SDF by linking the subjective discount parameter to an exogenously determined risk-free rate of interest via the key restriction

$$\gamma \log \beta_t = -y_t^{(1)} \tag{5}$$

where $y_t^{(1)}$ is the log yield on a 1-quarter bond; i.e. one period is a quarter in our discrete-time yield curve model. Note that (5) implies that β_t^{γ} takes values between zero and one, since it equals the price of the 1-quarter bond. As in Obstfeld (1990), our model with a variable rate of time preference implies that consumption and asset prices depend on a short-term rate of interest. The restriction in (5) is motivated here by the central role played by the short-term rate of interest in the determination of bond prices. Indeed, the short rate is a fundamental building block for yields of other maturities, which are just risk-adjusted averages of expected future short rates.[7]

[7] Equation (5) might give the impression that the model will admit arbitrage opportunities. We will see that the SDF in (4) with (5) coupled with an affine specification ensures that the resulting bond prices remain arbitrage-free.

Following Ang *et al*. (2006), our model is based entirely on observable factors which we collect in a state vector X_t. Both macroeconomic variables and yield curve factors are included in the state vector. Ang *et al*. argue that two yield curve factors are sufficient to model the dynamics of yields at the quarterly frequency. Following those authors, we use the short rate, $y_t^{(1)}$, as a measure of the level factor of the yield curve and the 5-year term spread, $y_t^{(20)} - y_t^{(1)}$, as a measure of the slope factor of the yield curve. The macroeconomic factors are collected along with two term structure factors in the state vector so that $X_t = (y_t^{(1)}, y_t^{(20)} - y_t^{(1)}, r_t, \pi_t, c_t)'$. As in Ang *et al*., the vector of state variables follows a first-order VAR process:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \tag{6}$$

where the errors are distributed according to a multivariate standard normal distribution.[8]

The time-*t* price of a nominal bond that pays one dollar at time $t + n$ is determined by the recursive relation

$$P(t, n) = E_t[m^{\$}_{t+1} P(t + 1, n - 1)]$$

with the boundary condition $P(t + n, 0) = 1$; i.e. the price of a nominal bond with instantaneous maturity equals one dollar.[9] The logarithm of the nominal SDF is then

$$\log m^{\$}_{t+1} = -y_t^{(1)} + J' X_{t+1} \tag{7}$$

where $J = (0, 0, \gamma - 1, -1, \alpha - \gamma)'$. We will see that an affine structure ensures the identification of the corresponding equations in the state VAR process when $n \geq 1$, even though the vector J contains some zeros. Note that when $n = 1$ the SDF in (7) will satisfy the usual relationship $r_t^f = 1/E_t[m^{\$}_{t+1}]$, where r_t^f is the gross risk-free rate of interest.

Bond prices are parameterized as exponential linear functions of the state vector so that

$$P(t, n) = \exp(A(n) + B(n)' X_t) \tag{8}$$

for a scalar $A(n)$ and a 5×1 vector $B(n)$ of coefficients that are functions of the time to maturity n. The boundary condition $P(t + n, 0) = 1$ implies that $A(0) = 0$ and $B(0) = \mathbf{0}$. General solutions for the coefficients in (8) are based on the assumption that $m^{\$}_{t+1} P(t + 1, n - 1)$ is conditionally log-normal and the associated moments:

$$\begin{aligned}
E_t[\log m^{\$}_{t+1}] &= -y_t^{(1)} + J'(\mu + \Phi X_t), \\
E_t[\log P(t + 1, n - 1)] &= A(n - 1) + B(n - 1)'(\mu + \Phi X_t), \\
\mathrm{var}_t[\log m^{\$}_{t+1}] &= J' \Sigma \Sigma' J, \\
\mathrm{var}_t[\log P(t + 1, n - 1)] &= B(n - 1)' \Sigma \Sigma' B(n - 1), \\
\mathrm{cov}_t[\log m^{\$}_{t+1}, \log P(t + 1, n - 1)] &= B(n - 1)' \Sigma \Sigma' J
\end{aligned}$$

[8] The assumption that Σ is time-invariant is arguably unrealistic. Our goal, however, is to assess whether equilibrium restrictions inferred from observed bond yields can improve a popular reduced-form no-arbitrage model which maintains that assumption.

[9] As n goes to 0, the gross rates (of consumption growth, market return, and inflation) appearing in the definition of the SDF go to 1. The preference restriction that ensures an instantaneous SDF value of 1 is $\beta_t = 1$; i.e. a zero rate of psychological time preference.

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Specifically, bond prices are given by (8) with coefficients $A(n)$ and $B(n)$ determined by the backward recursions

$$A(n+1) = A(n) + [J + B(n)]'\mu + \frac{1}{2}[J + B(n)]'\Sigma\Sigma'[J + B(n)],$$
$$B(n+1) = \Phi'[J + B(n)] - e_1 \tag{9}$$

where $e_1 = (1, 0, 0, 0, 0)'$. The boundary restrictions $A(0) = 0$, $B(0) = \mathbf{0}$ imply that the recursions must satisfy the initial conditions $A(1) = 0$ and $B(1) = -e_1$. The difference equations in (9) that determine $A(n)$ and $B(n)$ are derived by induction, as in Ang and Piazzesi (2003).

The inclusion of the two term structure factors $y_t^{(1)}$ and $y_t^{(20)} - y_t^{(1)}$ in the state vector implies that the model prices the 1- and 20-quarter bonds without error. The first set of these internal consistency constraints is given by the initial conditions for the recursive definitions of the coefficients $A(n)$ and $B(n)$. The second set of constraints is

$$A(20) = 0,$$
$$B(20) = -20(e_1 + e_2) \tag{10}$$

where e_i is a 5×1 vector of zeros with a 1 in the ith element. These constraints ensure that the 20-quarter yield is the sum of the first two factors in X_t. The other yields are then functions of $y_t^{(1)}$ and $y_t^{(20)} - y_t^{(1)}$ and the other factors included in X_t. The yields not included as factors are thus subject to a pricing error.

The second equation of the backward recursions in (9) features the product $J'\Phi$, which might give the impression that the short rate and the term spread cannot be identified via bond prices when $n \geq 1$. The initial conditions, however, ensure the identification of the short-rate equation in the state VAR process. To see that the term spread is also identified, note that when the persistence matrix Φ admits an inverse, the factor loadings can be written as a forward recursion:

$$B(n)' = [B(n+1) + e_1]'\Phi^{-1} - J' \tag{11}$$

with the terminal conditions in (10). This recursion is mathematically equivalent to the one in (9) and makes clear that the term-spread equation is statistically identified.

The bond pricing equation in (8), along with the coefficients in (9), provides a characterization of the entire yield curve. In particular, it describes the joint dynamics of bond yields of various maturities and the vector of state variables. The model-implied yield on a continuously compounded n-period zero-coupon bond, $Y(t, n) = -\log P(t, n)/n$, is an affine function of the state vector:

$$Y(t, n) = -\frac{A(n)}{n} - \frac{B(n)'}{n}X_t \tag{12}$$

From the bond pricing equation, the time-t model-implied forward rate which applies between times $t + n$ and $t + n + s$ ($s \geq 1$), $F(t, n, s) = (\log P(t, n) - \log P(t, n + s))/s$, can be computed as

$$F(t, n, s) = -\frac{[A(n+s) - A(n)]}{s} - \frac{[B(n+s) - B(n)]'}{s}X_t \tag{13}$$

and the short rate expected to prevail at time $t + n$ is given by

$$E_t[y_{t+n}^{(1)}] = e_1'\left[\sum_{i=1}^{n}\Phi^{n-i}\mu + \Phi^n X_t\right] \tag{14}$$

where Φ^0 is set equal to the 5×5 identity matrix.

The expectations hypothesis is a restriction on the risk premium in the relationship

$$F(t, n, 1) = E_t[y_{t+n}^{(1)}] + \mathrm{RP}(t, n)$$

where $\mathrm{RP}(t, n)$ is the time-t risk premium. From the difference equations in (9), we have $B(n+1)' - B(n)' = (J - e_1)'\Phi^n$. Thus the model-implied risk premium when $n \geq 1$ is

$$\mathrm{RP}(t, n) = -J'\Phi^n X_t + \text{constant} \tag{15}$$

The expression in (15) shows that in addition to the preference parameters in J, the matrix Φ also plays a crucial role for the risk premiums. Indeed, the risk premium is time-varying whenever $J'\Phi^n \neq 0$. In contrast, the risk premium is constant and the expectations hypothesis holds when $\Phi = 0$.

With the restriction $\alpha = \rho$, the model reduces to an expected utility model albeit except for the time-varying subjective discount parameter. The expected utility version implies a separable time-additive preference structure, since the short-term rate of interest is exogenous; i.e. the subjective discount parameter does not depend on consumption choices. In that case, the CRRA is the reciprocal of the EIS and the return on the market portfolio plays no contemporaneous role in the SDF. Even though the market return then becomes an unspanned risk factor it may still play a predictive role for bond yields; see Collin-Dufresne and Goldstein (2002) and Wright and Zhou (2009).

Since the market return appears in the SDF in the non-expected utility specification, it seems to imply that the model should also price the market portfolio without error. However, as already mentioned, using returns on an equity index as a proxy for the market portfolio creates a wedge between a market portfolio that pays off consumption and an equity index portfolio that pays off dividends. Therefore, if the goal was to find a model that prices equally well bonds and equities, we would need at least to add dividends in the state vector.[10] Our ambition is more modest. We want to assess whether equilibrium restrictions inferred from observed bond yields improve a popular reduced-form no-arbitrage model in matching stylized facts about bond yields and returns. In that respect, the presence of the market return plays two fundamental roles: (i) it captures the links between the bond and the equity markets induced by investors' portfolio decisions; and (ii) it disentangles investors' attitudes towards risk and intertemporal substitution. With this important remark in mind, the next section presents an empirical assessment of the equilibrium model and the role played by the macroeconomic factors in explaining bond risk premiums.

3. MODEL ESTIMATION AND EVALUATION

3.1. Benchmark Reduced-Form Model

The described equilibrium model links the dynamics of the term structure of interest rates to observable macroeconomic variables. Ang and Piazzesi (2003) also establish such a link through a reduced-form model of the term structure. For comparison, their approach is used here to derive a reduced-form bond pricing equation given the same specification of state variables used for the equilibrium-based model.

[10] Few models aim at pricing equities and bonds. The equity premium puzzle reflects the difficulty to price equities and the risk-free rate. Solutions to the puzzle (Campbell and Cochrane, 1999; Bansal and Yaron, 2004) have been extended to price the term structure of interest rates (Wachter, 2006; Bansal and Shaliastovich, 2009; respectively). However, those models are calibrated to fit stylized facts about the term structure.

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The approach assumes that the nominal SDF follows a conditionally log-normal process of the form

$$\log m_{t+1}^{\$} = -y_t^{(1)} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1} \tag{16}$$

where λ_t are time-varying market prices of risk. The vector λ_t is parametrized as an affine process:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{17}$$

so that λ_0 is a 5×1 vector and λ_1 is a 5×5 matrix. Equations (16) and (17) relate shocks in the state VAR process to the SDF and therefore determine how those factor shocks affect all yields. As before, bond prices take the exponential linear form in (8) and the solutions for the coefficients are based on the assumption that $m_{t+1}^{\$}P(t+1, n-1)$ is conditionally log-normal. In this case, the relevant moments are

$$\begin{aligned}
E_t[\log m_{t+1}^{\$}] &= -y_t^{(1)} - \frac{1}{2}\lambda_t'\lambda_t, \\
E_t[\log P(t+1, n-1)] &= A^{na}(n-1) + B^{na}(n-1)'(\mu + \Phi X_t), \\
\text{var}_t[\log m_{t+1}^{\$}] &= \lambda_t'\lambda_t, \\
\text{var}_t[\log P(t+1, n-1)] &= B^{na}(n-1)'\Sigma\Sigma' B^{na}(n-1), \\
\text{cov}_t[\log m_{t+1}^{\$}, \log P(t+1, n-1)] &= -B^{na}(n-1)'\Sigma\lambda_t
\end{aligned}$$

The implied no-arbitrage bond yields are given by

$$Y^{na}(t, n) = -\frac{A^{na}(n)}{n} - \frac{B^{na}(n)'}{n}X_t, \tag{18}$$

where the coefficients $A^{na}(n)$ and $B^{na}(n)$ are defined recursively by

$$\begin{aligned}
A^{na}(n+1) &= A^{na}(n) + B^{na}(n)'(\mu - \Sigma\lambda_0) + \frac{1}{2}B^{na}(n)'\Sigma\Sigma' B^{na}(n), \\
B^{na}(n+1) &= (\Phi - \Sigma\lambda_1)' B^{na}(n) - e_1
\end{aligned} \tag{19}$$

As before, we have restrictions of the form $A^{na}(1) = 0$, $B^{na}(1) = -e_1$ and $A^{na}(20) = 0$, $B^{na}(20) = -20(e_1 + e_2)$. See Ang and Piazzesi (2003) and Ang *et al*. (2006) for additional details.

The reduced-form model implies that the risk premium on an n-period bond is given by

$$\text{RP}^{na}(t, n) = e_1'[(\Phi - \Sigma\lambda_1)^n - \Phi^n]X_t + \text{constant} \tag{20}$$

which shows how λ_1 affects the risk premium through its interaction with Σ. Note that when $\lambda_1 = 0$, the term in square brackets in (20) becomes zero. In that case the risk premiums do not depend on X_t and become constant.

3.2. Estimation Methodology

The possible estimation procedures can be classified in two ways: (i) those that explicitly impose the state variable dynamics, and (ii) those that only use information about the state variable dynamics that is implicit in bond risk premiums. The maximum likelihood procedures of Chen

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and Scott (1993) and Joslin *et al*. (2011) fall in the first category. Those procedures assume that some yields cannot be perfectly priced and proceed by imposing a parametric distribution for the pricing errors. Ang *et al*. (2006) adopt a consistent two-step estimation procedure that also falls in the first category, but which does not make any distributional assumptions about the pricing errors. In the first step, estimates of μ, Φ, and Σ as they appear in (6) are found by ordinary least squares. In the second step, λ_0 and λ_1 are estimated taking as given the first-step VAR estimates. This is done by solving the nonlinear least-squares problem:

$$\min_{\{\lambda_0,\lambda_1\}} \sum_T \sum_N (y_t^{(n)} - Y^{na}(t,n))^2 \tag{21}$$

where $y_t^{(n)}$ is the market yield of an n-period bond at time t and $Y^{na}(t,n)$ is the corresponding model-implied yield; the first summation is over available time observations and the second summation is over the yields used to estimate the model. Here we followed the Ang *et al*. approach to estimate the reduced-form model. Specifically, the minimization in (21) was done with the Nelder–Mead simplex algorithm, and once the optimum was found the covariance matrix was estimated using numerical derivatives of the nonlinear regression function with respect to the vector of parameters.[11]

In the case of the equilibrium-based model, imposing the explicitly obtained estimates of μ, Φ, and Σ à la Ang *et al*. would only leave the two preference parameters in J to match the entire panel of yields data in the second step, which is obviously insufficient. Therefore, we follow the second estimation approach and use only the information about state variable dynamics implicit in bond risk premiums. From the expression in (15), one can see that the equilibrium risk premiums are completely determined by the two preference parameters in J and the 5×5 matrix Φ. This means that we can estimate the equilibrium-based model by solving the second-step nonlinear least-squares problem with respect to those parameters, taking as given only the first-step estimates of μ and Σ. Specifically, our second step solves

$$\min_{\{\mathrm{CRRA},\mathrm{EIS},\Phi\}} \sum_T \sum_N (y_t^{(n)} - Y(t,n))^2 \tag{22}$$

where $Y(t,n)$ is given by (12). Therefore, as with the reduced-form model, we let the bond market data tell us whether risk premiums in the equilibrium model are time-varying. Note that there is no reason to expect the estimate of Φ so obtained to resemble its counterpart in the explicitly backward-looking VAR. Indeed, the Φ backed out from (22) captures investors' anticipations about future values of the state variables implicitly contained in bond risk premiums.

Our approach is similar to that in Garcia *et al*. (2003), who estimate a consumption-based option pricing model. Their model has parameters that describe investor preferences and state variable dynamics, which in their case are those of aggregate consumption and dividends. The model is estimated by a method of moments using only the moments of stock returns and those of option prices. Since consumption is not explicitly used, it need not even be observable. Thus there again the parameter estimates capture the dynamics of the state variables in as much as they are implicitly contained in financial markets data.

The estimation of the equilibrium-based model and the reduced-form model involves about the same number of parameters in the second step: 27 for the non-expected utility model and 30 for

[11] As a further check, we also computed bootstrap standard errors by recursively generating data according to the VAR specification, then generating yields data from the bond pricing formulas, and finally estimating the model parameters using the simulated data. A bootstrap distribution was generated from 1000 replications of this (numerically intensive) procedure.

the reduced-form model. It is also important to note that the two specifications are not nested. This means that we would not expect the reduced-form model to necessarily provide a better fit to the data.

3.3. Modified Reduced-Form Model

In the two-step estimation procedure of the reduced-form model, the parameters λ_0 and λ_1 are estimated with no implications for μ, Φ, and Σ. On the other hand, the procedure for the equilibrium-based model treats Φ as a matrix of risk premium parameters and estimates them along with the two preference parameters in the second step. One can argue that we should level the playing field between these two ways of estimating Φ—explicitly from the state variables or from the bond yields. Therefore, we also consider a modified version of the reduced-form model in which the λ parameters are estimated conditional on the matrix Φ obtained from estimation of the non-expected utility model; i.e. by conditioning on the implicit AR parameter estimates. This will tell us if risk prices have a significant additional value in explaining the stylized facts about bond yields and returns.

3.4. Data Description

The macroeconomic fundamentals VAR model is estimated using data on US nominal interest rates, equities, inflation, and real consumption. Although the raw data are available at the monthly frequency, we follow Wachter (2006) and construct a quarterly dataset in order to reduce the influence of higher-frequency noise in inflation and short-term movements in interest rates.[12]

Real aggregate consumption is based on personal consumption expenditures on nondurables and services obtained from the Bureau of Economic Analysis. Per capita consumption is then obtained by dividing the real aggregate consumption by the total population. The level of the market portfolio is proxied using a value-weighted index of stocks, including dividends, traded on the NYSE, AMEX, and NASDAQ markets obtained from the Center for Research in Security Prices (CRSP). For inflation, we use data on the Consumer Price Index (CPI) obtained from the Federal Reserve Bank of St Louis. The level data on real per capita consumption, the stock index, and the CPI were aggregated up to the quarterly frequency by averaging the monthly observations. The return on the market portfolio, the rate of inflation, and the growth rate of consumption were then defined as the changes in the (log) values of the corresponding level data. The bond data are a set of monthly zero-coupon yields obtained from CRSP. These monthly yields were averaged to obtain quarterly yields on bonds with maturities of 1, 2, 4, 8, 12, 16, and 20 quarters. These data definitions ensure that the yields incorporate information about the rates of inflation, consumption growth, and market return throughout the quarter. The quarterly dataset has 182 observations from 1959 : Q3 to 2004 : Q4.

Table I provides summary statistics of the yield data at the quarterly frequency. As usual, the yield curve slopes upward on average. Further, the standard deviation, skewness, and kurtosis tend to be higher for shorter bond maturities.

[12] As Wachter states, higher-frequency interest rate fluctuations would seem difficult to explain using an equilibrium model with macroeconomic variables. Moreover, given the nonlinearity of the considered models, it was important to search over specified grids of initial values, which increased the computational cost involved. The depth of recursions when computing (9) and (19) with monthly data prohibits such a thorough exploration of the parameter space.

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Table I. Summary statistics of yields data

	Maturity in quarters						
	1	2	4	8	12	16	20
Mean	0.056	0.059	0.061	0.063	0.065	0.066	0.067
SD	0.028	0.028	0.027	0.027	0.026	0.025	0.025
Skewness	1.010	0.962	0.826	0.824	0.852	0.871	0.875
Kurtosis	4.474	4.298	3.886	3.712	3.664	3.597	3.478
Min.	0.009	0.010	0.011	0.014	0.017	0.022	0.025
Max.	0.151	0.159	0.155	0.154	0.151	0.150	0.145

Note: The quarterly dataset has 182 observations from 1959 : Q3 to 2004 : Q4.

Table II. Parameter estimates: equilibrium model

Panel A: Non-expected utility version					
Preference parameters					
CRRA	6.057				
	[5.393, 6.722]				
EIS	0.359				
	[0.297, 0.421]				
Matrix Φ					
Short rate	0.975	0.263	−0.024	0.101	0.284
	[0.346, 1.604]	[−0.355, 0.881]	[−0.576, 0.527]	[−0.498, 0.699]	[−0.298, 0.866]
Spread	0.021	0.804	0.028	−0.133	−0.337
	[−0.704, 0.747]	[0.120, 1.488]	[−0.742, 0.798]	[−0.835, 0.568]	[−1.103, 0.428]
Market return	0.136	1.067	0.126	−0.625	−1.526
	[−0.512, 0.785]	[0.346, 1.788]	[−0.542, 0.795]	[−1.215, −0.040]	[−2.130, −0.923]
Inflation	−0.020	0.402	0.042	0.641	0.372
	[−0.749, 0.708]	[−0.289, 1.092]	[−0.616, 0.700]	[0.017, 1.267]	[−0.247, 0.992]
Consumption	0.046	0.184	0.025	−0.242	−0.409
	[−0.291, 0.383]	[−0.228, 0.596]	[−0.283, 0.334]	[−0.578, 0.093]	[−0.718, −0.101]
Panel B: Expected utility version					
Preference parameters					
CRRA	2.747				
	[1.880, 3.614]				
EIS	0.364				
	[0.249, 0.478]				
Matrix Φ					
Short rate	1.007	0.182	0.009	−0.006	−0.063
	[0.457, 1.556]	[−0.352, 0.717]	[−0.421, 0.440]	[−0.560, 0.548]	[−0.686, 0.558]
Spread	−0.043	0.881	−0.038	−0.110	−0.396
	[−1.069, 0.981]	[0.006, 1.756]	[−0.796, 0.720]	[−1.007, 0.786]	[−1.463, 0.671]
Inflation	0.608	0.316	0.402	0.404	0.515
	[−0.326, 1.543]	[−0.588, 1.222]	[−0.530, 1.335]	[−0.573, 1.382]	[−0.474, 1.506]
Consumption	−0.226	−0.121	−0.156	−0.271	−0.449
	[−1.097, 0.645]	[−1.109, 0.867]	[−1.133, 0.819]	[−1.088, 0.545]	[−1.357, 0.459]

Note: CRRA, coefficient of relative risk aversion; EIS, elasticity of intertemporal substitution. The expected utility model (panel B) restricts the CRRA to the reciprocal of the EIS, so that the market return plays no contemporaneous role in the SDF. Numbers in square brackets are symmetric 95% confidence intervals. In the restricted case, the confidence limits for the EIS were found by the delta method.

3.5. Estimation Results

Estimation results for the equilibrium model are reported in Table II, along with 95% confidence intervals for each parameter. The table reports results for both the non-expected utility case and the expected utility case, where the CRRA is the reciprocal of the EIS. The point estimate for

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the CRRA in the unrestricted case is around 6, and is estimated quite precisely as seen from the narrow confidence interval. This finding is in line with those of Epstein and Zin (1991), Schwartz and Torous (1999), and Malloy *et al*. (2009).

Table II shows that the EIS is also estimated precisely, with a point estimate of 0.359. That value is consistent with the findings of Epstein and Zin (1991), who found the EIS to be statistically less than 1. Schwartz and Torous (1999) report a point estimate of 0.226 for the EIS. Their results corroborate the work of Hall (1988) and Campbell (1999), who conclude that the EIS is small and positive and statistically different from zero. More recently, Liu *et al*. (2008) find EIS estimates ranging from 0.07 to 0.15.

Looking at the estimate of Φ in the non-expected utility case, each of the state variables appears statistically significant for some element of the matrix. It is important to bear in mind that those estimates represent the persistence of state variables that is implicitly contained in the bond yields. The estimate of Φ under expected utility exhibits a very different pattern. In that case, the only significant elements are associated with the short rate and the term spread. The inflation rate, the return on the market portfolio, and consumption growth are nowhere significant. The fact that consumption makes no significant contribution to the explanation of bond yields provides yet more evidence against the consumption-based asset pricing model with power utility.

Table III reports the parameter estimates for the benchmark reduced-form model (panel A) and the modified version (panel B). The reported confidence intervals reveal that many of the parameter estimates have large standard errors, as is common in reduced-form factor models of the term structure. In panel A of Table III, the market return is the only variable that appears significant in the average market price of risk, λ_0. This result is interesting since Ang and Piazzesi (2006) find that such unconditional means are hard to pin down in small samples owing to the persistent nature of bond yields. On the other hand, each of the state variables plays some significant role in determining the time variation of market prices of risk. The significance of every element in the third column of λ_1 corresponding to the market return is worth noting. In sharp contrast, we see from panel B that all the λ parameters lose their statistical significance in the modified reduced-form model. This provides further support for the non-expected utility model, since the estimation of 30 additional parameters cannot significantly increase the fit with respect to the restricted $\hat{\Phi}$ implied by the non-expected equilibrium model.

Table IV reports summary statistics of the in-sample absolute pricing errors (in basis points). It is immediately clear that relaxing the expected utility constraint improves the fit of the equilibrium model. This result confirms that the market return plays a relatively important contemporaneous role in the pricing of bonds. The non-expected utility equilibrium model fares well against both versions of the reduced-form model, as seen from the small differences in pricing errors. The maximal pricing error in Table IV is only about 140 basis points, which occurs under the expected utility equilibrium model with 4-quarter bonds. Despite the relative differences across models, the pricing errors in Table IV show that these four specifications fit the data very well by any standard.

4. EMPIRICAL IMPLICATIONS

4.1. Volatilities of Yields and Yield Changes

Litterman *et al*. (1991) document a hump-shaped pattern in the term structure of unconditional volatilities of yields and yield changes. Panel A of Table V shows the volatilities of the actual market yields across maturities. Here the hump occurs at a maturity of 2 quarters: volatility is relatively lower for 1-quarter bonds, peaks for 2-quarter bonds, and then decreases monotonically as the maturity increases from 4 to 20 quarters. A similar pattern occurs in the term structure of unconditional volatilities of yield changes, shown in panel A of Table VI.

Table III. Parameter estimates: reduced-form model

	λ_0	λ_1				
Panel A: Benchmark version						
Short rate	−0.194	−12.340	−12.378	1.053	32.508	14.225
	[−3.451, 3.063]	[−14.857, −9.823]	[−15.821, −8.935]	[0.534, 1.573]	[29.025, 35.990]	[9.387, 19.062]
Spread	3.569	2.825	−13.052	12.491	6.406	−5.336
	[−4.013, 11.152]	[−4.089, 9.741]	[−20.963, −5.141]	[5.029, 19.951]	[−1.767, 14.580]	[−13.630, 2.958]
Market return	18.465	7.236	3.831	36.799	14.925	7.398
	[11.032, 25.897]	[−1.454, 15.928]	[−5.264, 12.926]	[28.188, 45.410]	[6.212, 23.638]	[−2.829, 17.627]
Inflation	3.425	−5.155	−30.101	10.808	3.845	14.739
	[−5.709, 12.559]	[−13.344, 3.032]	[−41.409, −18.791]	[1.353, 20.264]	[−6.883, 14.575]	[3.738, 25.741]
Consumption	−5.741	−4.640	−92.542	−16.065	−0.639	0.067
	[−14.795, 3.312]	[−12.479, 3.199]	[−102.075, −83.009]	[−24.614, −7.517]	[−9.417, 8.139]	[−9.072, 9.207]
Panel B: Modified version						
Short rate	−0.212	−2.997	10.605	−1.434	−1.268	−6.959
	[−2.908, 2.483]	[−5.707, −0.286]	[7.728, 13.482]	[−4.147, 1.277]	[−4.865, 2.328]	[−12.387, −1.674]
Spread	0.236	3.796	−12.896	3.008	1.089	1.821
	[−7.419, 7.892]	[−3.601, 11.194]	[−19.888, −5.904]	[−4.358, 10.375]	[−6.845, 9.023]	[−6.167, 9.810]
Market return	−1.805	−3.927	2.450	1.701	0.118	−16.855
	[−9.903, 6.292]	[−12.104, 4.249]	[−8.319, 13.219]	[−6.481, 9.883]	[−9.233, 9.470]	[−29.853, −3.857]
Inflation	4.589	7.778	7.308	3.397	−1.073	−0.152
	[−2.370, 11.549]	[−1.074, 16.631]	[−13.172, 27.789]	[−4.925, 11.720]	[−14.074, 11.927]	[−25.717, 25.413]
Consumption	1.885	−17.884	8.802	4.074	5.517	−0.108
	[−6.113, 9.885]	[−24.747, −11.020]	[−1.021, 18.626]	[−3.383, 11.532]	[−2.661, 13.697]	[−10.705, 10.487]

Note: Numbers in square brackets are symmetric 95% confidence intervals.

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Can any of the four model specifications reproduce the term structure of volatilities? To answer this question, we generated 1000 samples of artificial yields for each maturity of the same length as the actual data under each model specification. This involved using the OLS estimates to recursively generate data for the state variables according to the VAR specification and then feeding those data into the bond-pricing formulas, evaluated at the point estimates in Tables II and III, to generate the yields data.

The volatilities of the simulated yields and yield changes are reported in panels B and C of Tables V and VI, respectively. The reported statistics are the mean values across the 1000 replications, along with asymmetric 95% confidence intervals constructed from the quantiles of the simulated distributions. For both yields and yield changes, the non-expected utility model successfully reproduces the hump-shaped pattern of volatilities across bond maturities. Conversely, the expected utility and the reduced-form models do not reproduce the hump. Indeed, Tables V and VI show a strictly decreasing term structure of volatilities for those three specifications.

4.2. Violations of the Expectations Hypothesis

4.2.1 Campbell–Shiller Regressions

According to the expectations hypothesis of the term structure of interest rates, long-term yields are the average of expected future short yields over the holding period of the long-term asset, plus a constant risk premium. This implies that current spreads between yields of different maturities predict future yield changes. Campbell and Shiller (1991) consider predictive regressions of the

Table IV. In-sample absolute pricing errors (basis points)

	Maturity in quarters				
	2	4	8	12	16
Panel A: Equilibrium model					
Non-expected utility version					
Mean	13.20	19.94	16.36	11.91	7.45
SD	12.50	17.03	13.70	9.39	6.23
Min.	0.02	0.36	0.09	0.01	0.12
Max.	72.15	104.88	71.51	47.19	37.74
Expected utility version					
Mean	25.74	28.07	25.03	18.61	13.66
SD	18.38	22.14	19.47	14.75	11.85
Min.	0.25	0.31	0.25	0.01	0.11
Max.	87.70	139.46	112.66	70.98	64.55
Panel B: Reduced-form model					
Benchmark version					
Mean	14.55	20.56	16.93	13.72	12.36
SD	12.26	17.38	13.19	10.68	9.34
Min.	0.42	0.34	0.05	0.16	0.34
Max.	75.22	107.98	70.31	53.61	47.79
Modified version					
Mean	14.22	20.01	15.81	12.02	7.08
SD	11.93	16.84	13.31	9.39	5.77
Min.	0.09	0.13	0.07	0.26	0.02
Max.	74.89	109.61	82.76	53.78	26.91

Note: The absolute pricing errors are calculated over the 182 quarterly observations for each of the five maturities that are not assumed to be priced without any sampling error. The 1- and 20-quarter yields are priced without error. The expected utility model restricts the CRRA to the reciprocal of the EIS.

Table V. Volatilities of yields

n						
1	2	4	8	12	16	20
Panel A: Actual data						
2.799	2.830	2.776	2.704	2.613	2.563	2.524
Panel B: Equilibrium model						
Non-expected utility version						
2.501	2.535	2.462	2.332	2.272	2.239	2.198
[1.651, 3.704]	[1.652, 3.768]	[1.567, 3.736]	[1.434, 3.606]	[1.394, 3.557]	[1.368, 3.524]	[1.332, 3.486]
Expected utility version						
2.497	2.415	2.343	2.265	2.239	2.242	2.204
[1.604, 3.683]	[1.550, 3.571]	[1.466, 3.480]	[1.350, 3.412]	[1.321, 3.405]	[1.316, 3.441]	[1.299, 3.407]
Panel C: Reduced-form model						
Benchmark version						
2.490	2.481	2.429	2.349	2.296	2.248	2.197
[1.646, 3.642]	[1.623, 3.653]	[1.553, 3.617]	[1.458, 3.524]	[1.409, 3.461]	[1.382, 3.395]	[1.330, 3.308]
Modified version						
2.483	2.454	2.415	2.350	2.286	2.217	2.189
[1.623, 3.597]	[1.575, 3.580]	[1.527, 3.6560]	[1.443, 3.536]	[1.404, 3.482]	[1.356, 3.388]	[1.332, 3.344]

Note: The top panel reports the standard deviation of percentage yields. The next two panels show the same statistics implied by the four model specifications. The reported statistics are the mean values across 1000 bootstrap replications. Numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution.

Table VI. Volatilities of yield changes

n						
1	2	4	8	12	16	20
Panel A: Actual data						
0.850	0.852	0.809	0.725	0.651	0.605	0.567
Panel B: Equilibrium model						
Non-expected utility version						
0.860	0.864	0.786	0.662	0.613	0.594	0.575
[0.771, 0.955]	[0.773, 0.954]	[0.705, 0.868]	[0.594, 0.733]	[0.553, 0.679]	[0.534, 0.657]	[0.518, 0.637]
Expected utility version						
0.861	0.847	0.770	0.664	0.611	0.596	0.574
[0.775, 0.944]	[0.760, 0.934]	[0.693, 0.849]	[0.599, 0.734]	[0.551, 0.674]	[0.532, 0.659]	[0.513, 0.632]
Panel C: Reduced-form model						
Benchmark version						
0.863	0.841	0.773	0.678	0.641	0.636	0.573
[0.771, 0.952]	[0.752, 0.930]	[0.693, 0.852]	[0.611, 0.751]	[0.575, 0.706]	[0.572, 0.702]	[0.516, 0.631]
Modified version						
0.865	0.842	0.771	0.675	0.625	0.595	0.576
[0.772, 0.955]	[0.751, 0.927]	[0.688, 0.851]	[0.601, 0.748]	[0.556, 0.693]	[0.528, 0.659]	[0.511, 0.639]

Note: The top panel reports the standard deviation of percentage yield changes. The next two panels show the same statistics implied by the four model specifications. The reported statistics are the mean values across 1000 bootstrap replications. Numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution.

form

$$y_{t+1}^{(n-1)} - y_t^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} \frac{1}{n-1}(y_t^{(n)} - y_t^{(1)}) + \varepsilon_{t+1}^{(n)} \tag{23}$$

which should produce a slope coefficient of 1 under the expectations hypothesis. Campbell and Shiller find that the slope coefficient is less than 1 and decreasing in n. Bansal and Zhou (2002) show that this predictability evidence can be explained by a term structure model with regime shifts in the short rate and the market prices of risks.

We want to find out whether any of the term structure models we consider can generate the required risk premiums for the specific set of parameter values that correctly fit the data. Table VII shows the results for the regression in (23) with $n = 4, 8, 12, 16, 20$.[13] Panel A shows the slope coefficients and the R^2s found in the actual data. As in previous studies, the slope coefficients are negative and decreasing with maturity.

Panels B and C of Table VII show how closely the four models can mimic the pattern of slope coefficients. Following Bansal and Zhou (2002) and Wachter (2006), we generated 1000 samples of artificial yields, as described above for the term structure of volatilities. For each simulated sample, we ran the regression in (23) and computed the R^2. Table VII reports the mean slope coefficients along with asymmetric 95% confidence intervals. The non-expected utility model produces mean slope coefficients with the downward pattern across maturity and the actual coefficients are well

Table VII. Predictability of yield changes using yield spreads

	n				
	4	8	12	16	20
Panel A: Actual data					
$\beta_1^{(n)}$	−0.603	−1.019	−1.438	−1.685	−1.839
R^2	0.009	0.017	0.027	0.032	0.033
Panel B: Equilibrium model					
Non-expected utility version					
$\beta_1^{(n)}$	−0.135	−0.482	−0.963	−1.396	−1.873
	[−1.258, 1.139]	[−1.549, 0.767]	[−2.179, 0.401]	[−2.793, 0.115]	[−3.391, −0.252]
R^2	0.006	0.011	0.020	0.027	0.037
	[0, 0.029]	[0, 0.050]	[0, 0.075]	[0, 0.083]	[0, 0.105]
Expected utility version					
$\beta_1^{(n)}$	−0.937	−0.747	−1.341	−2.061	−1.896
	[−1.823, 0.006]	[−1.891, 0.409]	[−2.615, −0.116]	[−3.467, −0.718]	[−3.451, −0.340]
R^2	0.017	0.012	0.027	0.050	0.038
	[0, 0.056]	[0, 0.048]	[0, 0.084]	[0.004, 0.126]	[0.001, 0.108]
Panel C: Reduced-form model					
Benchmark version					
$\beta_1^{(n)}$	0.452	−0.357	−0.970	−1.649	−1.901
	[−0.675, 1.747]	[−1.497, 0.805]	[−2.295, 0.259]	[−3.214, −0.283]	[−3.551, −0.379]
R^2	0.008	0.008	0.019	0.032	0.038
	[0, 0.038]	[0, 0.042]	[0, 0.072]	[0, 0.096]	[0.002, 0.112]
Modified version					
$\beta_1^{(n)}$	0.353	−0.423	−0.992	−1.457	−1.867
	[−0.867, 1.733]	[−1.617, 0.931]	[−2.302, 0.395]	[−2.937, 0.064]	[−3.547, −0.210]
R^2	0.002	0.004	0.015	0.024	0.032
	[0, 0.035]	[0, 0.041]	[0, 0.071]	[0, 0.089]	[0, 0.105]

Note: The top panel reports the estimated slope coefficients and R^2s in the predictive regression of yield changes on yield spreads in (23). The next two panels show the same statistics implied by the four model specifications. The reported coefficients are the mean values across 1000 bootstrap replications. Numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution. Values less than 10^{-3} are reported as zero.

[13] As usually done, the change $y_{t+1}^{(n)} - y_t^{(n)}$ is used instead of $y_{t+1}^{(n-1)} - y_t^{(n)}$, since $y_{t+1}^{(n-1)}$ is not available. Bekaert *et al.* (1997) discuss the effects of this approximation.

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covered by the respective confidence intervals. In the expected utility version of the equilibrium model, however, the mean slope coefficients do not decrease monotonically with n, although the actual coefficients are covered by the respective confidence intervals. Perhaps a more serious problem revealed by Table VII is the slope coefficient associated with $n = 4$ in the benchmark reduced-form model (panel C, first column). In that case, the mean slope coefficient is positive and the actual slope coefficient of -0.603 is only marginally covered by the confidence interval $[-0.675, 1.747]$. The confidence interval $[-0.867, 1.733]$ in the bottom portion of panel C shows that the modified version of that model offers an improvement. This indicates that there might be a deeper problem with the risk premiums implied by the benchmark reduced-form model. We examine this next by looking at predictive regressions of excess bond returns using forward rates.

4.2.2 Cochrane–Piazzesi Regressions

Another way to state the expectations hypothesis of the term structure of interest rates is that holding-period excess returns should not be predictable. Cochrane and Piazzesi (2005) consider the predictive regression of 4-quarter excess bond returns on the initial yield and forward rates:

$$rx_{t+4}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} y_t^{(4)} + \sum_{i=2}^{5} \beta_i^{(n)} f_t^{(4i)} + \varepsilon_{t+4}^{(n)}, \quad n = 8,\ 12,\ 16,\ \text{and } 20 \tag{24}$$

where $rx_{t+4}^{(n)} = p_{t+4}^{(n-4)} - p_t^{(n)} - y_t^{(4)}$ is the return (in excess of the 4-quarter bond yield) from buying an n-quarter bond at time t and selling it as an $(n-4)$-quarter bond at time $t+4$, and $f_t^{(n)} = p_t^{(n-4)} - p_t^{(n)}$ is the forward rate for loans between time $t+n-4$ and $t+n$. Here $p_t^{(n)}$ is the log price of an n-year bond at time t. Note that time increments are in years. Cochrane and Piazzesi find a robust tent-shaped pattern of slope coefficients for all maturities, with regression R^2 values around 35%. This violation of the expectations hypothesis extends the classic regressions of Fama and Bliss (1987) and Campbell and Shiller (1991). Fama and Bliss found that the spread between the n-year forward rate and the 1-year yield predicts the 1-year excess return of the n-year bond, with R^2 about 18%. As mentioned above, Campbell and Shiller found similar results forecasting yield changes with yield spreads. Cochrane and Piazzesi's findings substantially strengthen that evidence against the expectations hypothesis. In particular, they show that the same linear combination of forward rates—the regressors in (24)—predicts bond returns at all maturities, while Fama and Bliss and Campbell and Shiller relate each bond's expected excess return to a different forward spread or yield spread.

The size of the predictability and nature of projection coefficients in regressions like (24) is quite puzzling and, as Bansal *et al.* (2004) state, 'constitutes a serious challenge to term structure models'. Bansal *et al.* account for the predictability evidence from the perspective of latent factor term structure models. They show that the regime-switching model of Bansal and Zhou (2002) can empirically account for these challenging features of the data, while affine specifications cannot. In this section, we ask whether the risk premiums generated by our model (based on observable factors) can also account for the tent-shaped predictability pattern.[14] To preview the results, it is only the non-expected utility model and the modified version of the reduced-form model that can do so. Both the expected utility version of the equilibrium model and the benchmark reduced-form model fail to account for these important features.

[14] The question is not whether one can construct market prices of risk that generate the return regressions in an affine model. Cochrane and Piazzesi (2005) show exactly how that can be done. As with the Campbell–Shiller regressions, the question we ask is whether any of the considered term structure models can generate the required risk premiums given the specific set of parameter values that correctly fit the data.

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Table VIII. Predictability of excess returns using forward rates

n	$\beta_0^{(n)}$	$\beta_1^{(n)}$	$\beta_3^{(n)}$	$\beta_5^{(n)}$	R^2
Panel A: Actual data					
8	−0.060	−0.900	2.159	−1.045	0.335
12	−0.089	−1.823	4.408	−2.287	0.362
16	−0.128	−2.631	6.132	−3.101	0.378
20	−0.168	−3.216	7.139	−3.438	0.366
Panel B: Equilibrium model					
Non-expected utility version					
8	−0.057	−1.354	3.291	−1.737	0.257
	[−0.139, −0.005]	[−2.349, −0.258]	[0.168, 6.072]	[−3.712, 0.411]	[0.080, 0.446]
12	−0.105	−2.547	6.197	−3.316	0.268
	[−0.246, −0.014]	[−4.305, −0.694]	[0.845, 11.117]	[−6.770, 0.414]	[0.096, 0.455]
16	−0.151	−3.616	8.866	−4.792	0.265
	[−0.352, −0.024]	[−6.107, −1.098]	[1.343, 15.896]	[−9.622, 0.471]	[0.100, 0.446]
20	−0.214	−4.630	10.446	−5.210	0.287
	[−0.482, −0.048]	[−7.803, −1.379]	[0.797, 19.549]	[−11.551, 1.564]	[0.113, 0.482]
Expected utility version					
8	−0.051	−0.458	0.972	−0.303	0.210
	[−0.122, −0.004]	[−0.916, 0.026]	[0.345, 1.556]	[−0.624, 0.046]	[0.071, 0.380]
12	−0.102	−1.139	2.137	−0.609	0.260
	[−0.225, −0.017]	[−1.982, −0.257]	[1.004, 3.214]	[−1.181, 0.005]	[0.106, 0.442]
16	−0.146	−1.953	3.405	−0.918	0.297
	[−0.324, −0.031]	[−3.115, −0.686]	[1.784, 4.908]	[−1.729, −0.060]	[0.136, 0.485]
20	−0.188	−2.385	3.197	−0.190	0.270
	[−0.414, −0.040]	[−3.836, −0.739]	[1.162, 5.107]	[−1.237, 0.907]	[0.099, 0.466]
Panel C: Reduced-form model					
Benchmark version					
8	−0.083	−0.498	0.586	0.204	0.241
	[−0.151, −0.031]	[−0.901, −0.032]	[−0.041, 1.187]	[−0.261, 0.687]	[0.074, 0.420]
12	−0.132	−1.045	1.255	0.267	0.246
	[−0.261, −0.040]	[−1.760, −0.241]	[0.039, 2.351]	[−0.569, 1.124]	[0.077, 0.418]
16	−0.184	−1.953	3.405	−0.918	0.297
	[−0.376, −0.057]	[−2.633, −0.474]	[0.399, 3.640]	[−1.055, 1.334]	[0.079, 0.415]
20	−0.262	−2.032	1.700	1.163	0.263
	[−0.514, −0.102]	[−3.293, −0.560]	[−0.424, 3.672]	[−0.422, 2.664]	[0.088, 0.435]
Modified version					
8	−0.052	−0.892	2.774	−1.670	0.263
	[−0.140, 0.005]	[−1.478, −0.300]	[0.718, 4.936]	[−3.515, 0.023]	[0.093, 0.458]
12	−0.081	−1.871	5.515	−3.340	0.275
	[−0.232, 0.023]	[−2.918, −0.839]	[1.910, 9.331]	[−6.715,−0.275]	[0.103, 0.468]
16	−0.097	−2.813	8.065	−4.915	0.280
	[−0.305, 0.045]	[−4.275, −1.338]	[2.959, 13.381]	[−9.587,−0.615]	[0.107, 0.475]
20	−0.159	−3.492	9.298	−5.306	0.291
	[−0.426, 0.019]	[−5.339, −1.580]	[2.894, 16.192]	[−11.185, 0.078]	[0.124, 0.490]

Note: The top panel reports the estimated coefficients and R^2s in the predictive regression of excess bond returns on forward rates in (24). The next two panels show the same statistics implied by the four model specifications. The reported coefficients are the mean values across 1000 bootstrap replications. Numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution.

Estimation results for the regressions in (24) are reported in the top panel of Table VIII. Consistent with the findings of Bansal *et al*. (2004), we also found that the use of the five forward rates in (24) creates a near-perfect collinearity problem in our dataset, so we concentrate on the regressions with $y_t^{(4)}$, $f_t^{(12)}$, and $f_t^{(20)}$ as regressors. The tent-shaped finding of Cochrane and Piazzesi (2005) is apparent in Figure 1, which plots the estimated regression coefficients. In

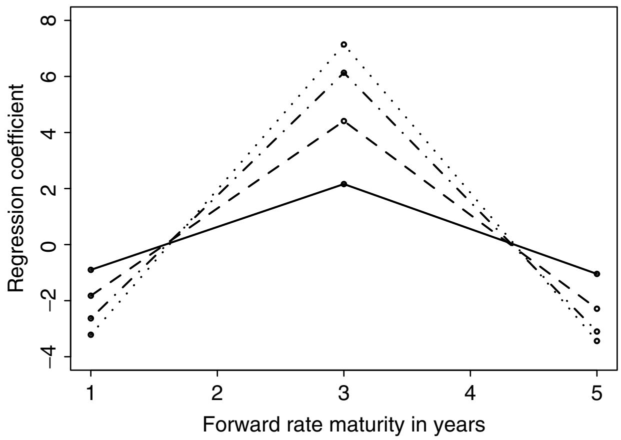

Figure 1. Predictability regression coefficients in the observed market data

Table VIII, we see that when the 8-quarter excess return is the regressand, the R^2 is around 34%, and that value reaches nearly 38% when the 16-quarter excess return appears as regressand.

Panels B and C of Table VIII show how closely the four models can mimic the tent-shaped pattern of regression coefficients. Following Bansal *et al.* (2004), we generated 1000 samples of the same length as the actual data for each model. As with the Campbell–Shiller regressions, this involved using the OLS estimates to recursively generate data for the state variables according to the VAR specification and then feeding those data into the bond-pricing formulas, evaluated at the point estimates in Tables II and III, to generate the yields data. For each simulated sample, we ran the regression in (24) and computed the R^2. Table VIII reports the mean regression coefficients along with asymmetric 95% confidence intervals. Figure 2 shows plots of the mean regression coefficients for the non-expected utility model (upper left), the expected utility model (upper right), the reduced-form model (lower left), and the modified reduced-form model (lower right). From those plots, it is immediately clear that only the non-expected utility model and the modified reduced-form model can empirically account for the tent-shaped pattern of coefficients from predictive regressions of excess bond returns on forward rates. The upper right plot of Figure 2 shows that the expected utility model fails to capture the predictability of the 3- and 5-year forward rate for all excess returns. As the lower left plot of Figure 2 shows, the benchmark reduced-form model fails even more so at capturing those predictability components. These shortcomings are further confirmed when sampling error is accounted for. The confidence intervals in Table VIII show more formally the correspondence between the non-expected utility model, the modified reduced-form models, and the actual data. In those two cases, all the actual coefficients are covered by the respective confidence intervals. Conversely, the confidence intervals for both the expected utility model and the benchmark reduced-form model fail to cover several of the actual coefficients. In particular, all the coefficients associated with the 3- and 5-year forward rates ($\beta_3^{(n)}$ and $\beta_5^{(n)}$) are not covered by the respective confidence intervals derived under those two specifications.

The predictability results presented here can be related to those obtained by Bansal *et al.* (2004). They show that their preferred two-factor regime-switching specification captures business cycle movements between economic expansions and recessions, and that these transitions affect the term structure of interest rates. A recession usually means a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP. It is therefore not surprising that our equilibrium model featuring inflation and consumption—an important component of GDP—also justifies the size and nature of bond return predictability. What

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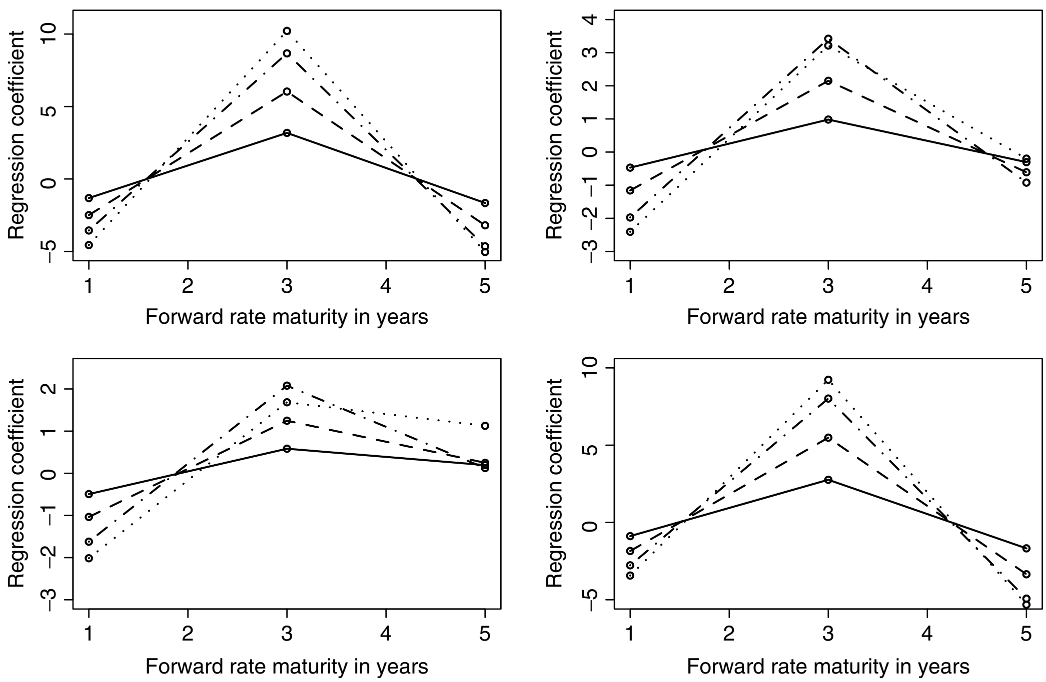

Figure 2. Predictability regression coefficients implied by the non-expected utility model (upper left), the expected utility model (upper right), the reduced-form model (lower left), and the modified reduced-form model (lower right)

is more intriguing are the contrasts between the non-expected utility model and the benchmark reduced-form model.

4.3. Understanding the Differences

Why are the implied risk premiums so different? A comparison of the coefficients in (9) with those in (19) provides some hints. Aside from the presence of the vector J in (9), the most notable difference between the two specifications is that the reduced-form model has $\mu - \Sigma\lambda_0$ and $\Phi - \Sigma\lambda_1$ in (19) instead of just μ and Φ, respectively, in (9). This means that the effects of μ and Φ on bond yields cannot be disentangled from that of Σ. This is also evident when comparing the expressions for the risk premiums in (15) and (20). Figure 3 plots the intercept and factor loadings for maturities ranging from 1 to 20 quarters, where the solid lines correspond to the non-expected utility model, the dashed lines to the expected utility model, the dotted lines to the reduced-form model, and the dot-dashed lines to the modified reduced-form model. By construction, the intercept and factor loadings are identical in value at the beginning and end points. Both the short rate (upper right) and the term spread (middle left) load in similar fashions across the four specifications. Using the non-expected utility model as the reference for comparisons, we see that the tight link between μ and Φ and Σ in the benchmark reduced-form specification leads to marked differences for the intercept terms (upper left), the market return loadings (middle right), the inflation loadings (lower left), and the consumption loadings (lower right). In those cases, we see a build-up effect as n increases. Interestingly, the modified version of the reduced-form model produces intercept and factor loadings much closer to the reference ones. Consider next the expected and non-expected utility models. The obvious difference is that the return on the market portfolio plays no contemporaneous role in the SDF under the expected utility specification and that expected utility restriction appears most noticeably in terms of the consumption loadings, especially for longer bond maturities (lower right).

Another related and important difference between the non-expected utility model and the benchmark reduced-form one can be seen from an examination of the innovations to their respective (log) SDFs, $\log m^{\$}_{t+1} - E_t[\log m^{\$}_{t+1}]$. The time series of implied innovations for the non-expected utility model are shown in the top plot of Figure 4 and those for the benchmark reduced-form model are shown in the middle plot of that figure; the plots are shown on the same scale. A striking result is the difference between the volatilities of the innovations. Indeed, the reduced-form SDF innovations appear far more volatile than those of the non-expected utility model. This clearly illustrates why the parameter estimates for the benchmark reduced-form model (in Table III) have large standard errors. It also explains the behavior of its factor loadings. The bottom plot of Figure IV shows the time series of innovations to the SDF of the modified reduced-form model. Relative to the pattern in the middle plot of that figure for the original reduced-form SDF, the modification achieves a remarkable reduction in overall volatility.

The coefficients of the predictive regression in (24) are further analyzed by decomposing their matrix form $\hat{\beta}^{(n)} = (X'X)^{-1}X'Y^{(n)}$, $n = 8$, 12, 16, and 20. Table IX shows the mean values of the matrix $(X'X)^{-1}$ and the vectors $X'Y^{(n)}$ across the 1000 bootstrap replications of the non-expected utility model and the reduced-form model. Using the non-expected utility model as the reference for comparison, we see that the benchmark reduced-form model generates initial yields $y_t^{(4)}$ and forward rates $f_t^{(12)}$, $f_t^{(20)}$ that are too volatile so that the associated $(X'X)^{-1}$ matrix is too 'small'. This is readily seen in the lower left plot of Figure 2, where the 5-year coefficients are compressed around zero.

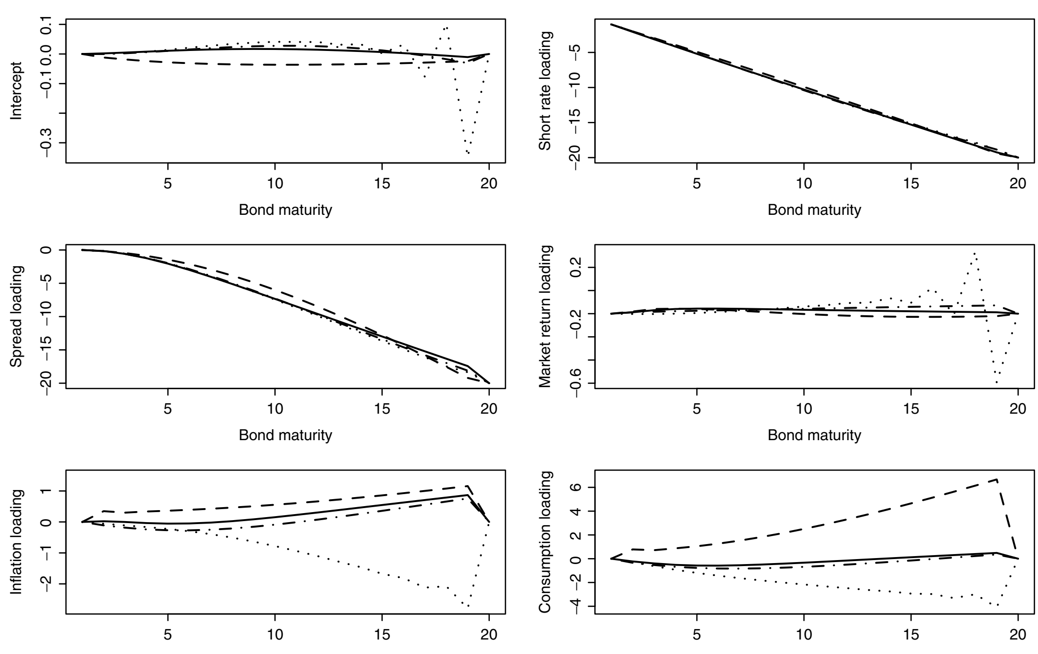

Figure 3. Intercept and factor loadings: the solid lines correspond to the non-expected utility model, the dashed lines to the expected utility model, the dotted lines to the reduced-form model, and the dot-dashed lines to the modified reduced-form model

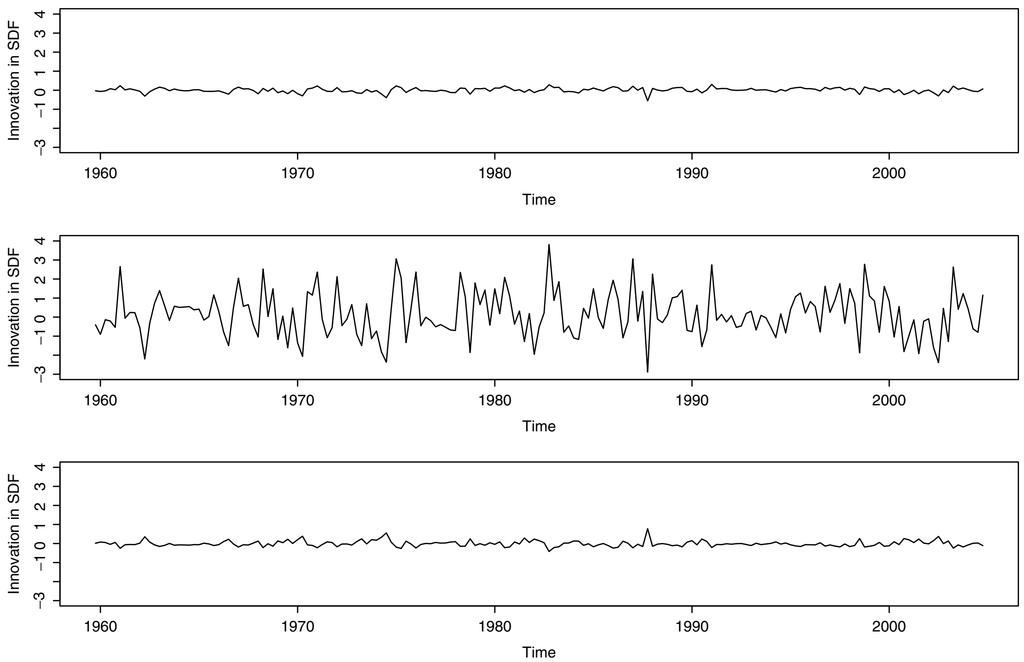

Figure 4. Innovation to the SDF of the non-expected utility model (top), the reduced-form model (middle), and the modified reduced-form model (bottom)

Table IX. Decomposition of predictability coefficients

$(X'X)^{-1}$				$X'Y^{(8)}$	$X'Y^{(12)}$	$X'Y^{(16)}$	$X'Y^{(20)}$
Non-expected utility model							
0.081	−0.221	0.890	−0.947	3.161	4.647	5.423	5.626
	32.967	−97.163	66.505	0.843	1.249	1.504	1.473
	307.104	−216.936	1.087	1.658	2.035	2.208	
		156.265	1.158	1.776	2.186	2.438	
Reduced-form model							
0.075	0.101	−0.142	−0.204	1.950	5.383	5.774	5.007
	4.646	−6.189	1.743	0.629	1.455	1.533	1.314
	15.637	−9.647	0.843	1.930	2.163	2.069	
		8.869	0.866	1.974	2.210	2.254	
Modified reduced-form model							
0.097	−0.301	1.888	−1.911	2.808	4.484	5.803	5.207
	10.747	−38.860	29.594	0.862	1.255	1.476	1.410
	189.214	−157.299	1.094	1.699	2.114	2.181	
		134.392	1.117	1.750	2.192	2.274	

Note: Entries represent the decomposition of the estimated coefficients in the predictive regression of excess bond returns on forward rates in (24), expressed here in matrix form. The reported coefficients are the mean values across 1000 bootstrap replications.

The efficiency gains achieved by the modified reduced-form model are apparent in the decomposition of the predictability coefficients, shown in the bottom portion of Table IX. The associated $(X'X)^{-1}$ matrix is now much closer to that of the non-expected utility model and indeed, as Figure 2 shows, the modified reduced-form gets much closer to producing the tent-like pattern of predictability regression coefficients. These results are perhaps not so surprising since the conditioning values of Φ, which affect the second-step λs, are precisely those that rationalize the risk premiums in the equilibrium model.

5. CONCLUSION

We have proposed an equilibrium model of the term structure of interest rates that can account for several stylized facts. In particular, this state-dependent recursive utility model can empirically account for the tent-shaped pattern and magnitude of coefficients from predictive regressions of excess bond returns on forward rates, documented by Cochrane and Piazzesi (2005). This is an important result since the equilibrium model ties the predictable variation in excess bond returns to underlying macroeconomic fundamentals and captures well the important features of bond risk premiums with economically plausible values for the structural preference parameters. The results emphasize the importance of both non-expected utility preferences and the variable rate of time preference for explaining violations of the expectations hypothesis.

Our empirical assessment reveals also that the equilibrium model can fit the term structure of interest rates as well as the arbitrage-free model. Our findings point to the fact that arbitrage-free models produce SDFs that are too volatile and therefore cannot explain bond risk premiums as well. They fail to account for the constraints that investors' preferences may impose between the prices-of-risk factors. Our main contribution is to show that these constraints matter empirically.

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