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Measuring High-Frequency Causality Between Returns, Realized Volatility, and Implied Volatility

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ABSTRACT
We provide evidence on two alternative mechanisms of interaction between returns and volatilities: the *leverage effect* and the *volatility feedback effect*. We stress the importance of distinguishing between

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realized volatility and *implied volatility* and find that *implied volatilities* are essential for assessing the *volatility feedback effect*. We also study the impact of news on returns and volatility. We introduce a concept of news based on the difference between implied and realized volatilities (the *variance risk premium*) and find that a positive variance risk premium has more impact on returns than a negative variance risk premium. (*JEL:* G1, G12, G14, C1, C12, C15, C32, C51, C53)

KEYWORDS: causality measure, implied volatility, leverage effect, realized volatility, variance risk premium

One of the many stylized facts about equity returns is an asymmetric relationship between returns and volatility. Volatility tends to rise following negative returns and falls following positive returns. Two main explanations for volatility asymmetry have been proposed in the literature. The first one is the *leverage effect*. While the term was originally coined with respect to financial leverage of a firm (see Black 1976; Christie 1982), it refers today to a negative correlation between lagged returns and current volatility.[1] The second explanation is the *volatility feedback effect*, which is related to a time-varying risk premium: if volatility is priced, an anticipated increase in volatility raises the required rate of return, implying an immediate stock price decline in order to allow for higher future returns (see Pindyck 1984; French, Schwert, and Stambaugh 1987; Campbell and Hentschel 1992; Bekaert and Wu 2000).

In this paper, we provide new evidence on these two interaction mechanisms between returns and volatilities by considering causality measures on high-frequency data. We also stress the importance of distinguishing between *realized volatility* and *implied volatility* when studying leverage and volatility feedback effects, and we find that *implied volatilities* are essential for detecting and assessing the *volatility feedback effect*.

On noting that the two explanations involve different causal mechanisms (see Bekaert and Wu 2000; Bollerslev, Litvinova, and Tauchen 2006), which may differ through both their direction and the time lags involved, we study the issue using short- and long-run causality measures recently introduced in Dufour and Taamouti (2010). The causality measures allow us to study and test the asymmetric volatility phenomena at several horizons. When considering horizons longer than one period, it is important to account for indirect causality. Auxiliary variables can transmit causality between two variables of interest at horizons strictly higher than one, even if there is no causality between the two variables at the horizon

[1] The concept of leverage effect was introduced to explain this negative correlation by the fact that a decrease in the price of a firm increases financial leverage and the probability of bankruptcy, making the asset riskier, hence an increase in volatility. Today the concept of dynamic leverage effect applies directly to stock market indices, without any rooting in changes of financial leverage (see Bouchaud, Matacz, and Potters 2001; Brandt and Kang 2004; Jacquier, Polson, and Rossi 2004; Ludvigson and Ng 2005; Bollerslev, Litvinova, and Tauchen 2006; Bollerslev, Sizova, and Tauchen 2009a).

one (see Dufour and Renault 1998; Dufour, Pelletier, and Renault 2006). Using high-frequency data increases the chance to detect causal links since aggregation may make the relationship between returns and volatility simultaneous. By relying on realized volatility measures, we avoid the need to specify a volatility model.

To be more explicit on the *causality* issue involved, the leverage effect explains why a negative return shock leads to higher subsequent volatility, while the volatility feedback effect explains how an anticipated increase in volatility may result in a negative return. Thus, volatility asymmetry may result from various causal links: from returns to volatility, from volatility to returns, instantaneous causality. Causality here is defined as in Granger (1969): a variable Y causes a variable X if the variance of the forecast error of X obtained by using the past of Y is smaller than the variance of the forecast error of X obtained without using the past of Y.

Concerning terminology, it is worthwhile pointing out that some authors may prefer to use terms like "predictibility" or "linear predictibility," instead of "causality." There is, however, a long philosophical tradition that reduces the concept of "causality" to the notion of "predictibility." This tradition goes back at least to Hume (1748) and includes numerous authors such as Carnap (1966), Feigl (1953), Salmon (1984), and Eells (1991). Whether there can be an empirically meaningful notion of causality that goes beyond a notion based on predictibility remains highly debatable. Here we take the view that "causality" can only be defined with respect to a particular model (e.g., a vector autoregressive [VAR] model) that involves specifying a set of variables some of which are classified as "endogenous" and other ones as "predetermined." In order to study "causation" issues with empirical data, it is necessary to specify a limited information set—a point quite explicit in Dufour and Renault (1998). There is no "absolute" (model-free) causation. This means that causality properties may change as the information set is modified (which includes changing data frequency and aggregation).

Further, the clearest criterion for classifying a variable as "predetermined" at a given date is the fact that it can be viewed as determined in the past (on the basis of the principle that the future cannot cause the past). Causality is then a predictibility property of the "endogenous" variables by "predetermined" variables. The notions of "causality" introduced by Wiener (1956) and Granger (1969) as well as their variants provide operational definitions of causality based on these ideas. Occasionally, a property of "Granger causality (or noncausality)" may be interpreted as "spurious," but this simply means that a different model or information set is considered. Such a situation illustrates the fact that "causality" can only be defined with respect to a given model and information set. Irrespective of the latter, one can always argue that "hidden" variables are driving the system, so variables that appear to Granger-cause other ones are simply reflecting "expectations" driven by hidden variables. This can easily be the case in finance and macroeconomics, where expectations typically constitute unobservable variables. The investigator may try to sort this out by introducing such unobserved variables (if reasonable measures or proxies can be obtained): this amounts to enlarging the information set, and ours may in turn be used with the new information. Note, however, that the "hidden variable criticism" may endlessly be reapplied since empirically usable information sets are always finite. In any case, demonstrating a Granger-causal structure provides useful information because it shows that either a "mechanism" or an "expectation phenomenon" is sufficiently important to allow forecasting. Further, in financial markets, expectations often determine actions (such as investment decisions) and so may have "effects" that go far beyond the mind-set of financial actors.[2]

In this paper, we stress that statistical tests of the null hypothesis of noncausality (in the sense of Wiener–Granger) constitute a poor way of analyzing causal structures. For example, we can distinguish between causal directions (from X to Y, from Y to X, instantaneous causality) and causality at different horizons. Different causality relations may coexist, but their relative importance may greatly differ. This suggests of finding means to quantify their degrees. Causality tests fail to accomplish this task, because they only provide evidence on the presence or the absence of causality, and statistical significance depends on the available data and test power. A large effect may not be statistically significant (at a given level), and a statistically significant effect may not be "large" from an economic viewpoint (or more generally from the viewpoint of the subject at hand) or relevant for decision making. [3]

In order to quantify and compare the strength of dynamic leverage and volatility feedback effects, we propose to use VAR models of returns and various measures of volatility at high frequency together with short- and long-run causality measures in Dufour and Taamouti (2010). For further discussion of the usefulness of causality measures and what they accomplish beyond Granger causality tests, we refer the reader to Dufour and Taamouti (2010).

Using five minute observations on S&P 500 Index futures contracts, we first consider causality measures based on a bivariate VAR involving returns and realized volatility. In this setting, we find a weak dynamic leverage effect for the first four hours in hourly data and a strong dynamic leverage effect for the first three days in daily data. The volatility feedback effect appears to be negligible, irrespective of the horizon considered.

Recently, using high-frequency data and simple *correlations*, Bollerslev, Litvinova, and Tauchen (2006) found an important negative correlation between volatility and (current and lagged) returns lasting for several days, while correlations between returns and lagged volatility are all close to zero. We differ from their study by using short- and long-run causality measures to quantify causality at different horizons. The difference between simple correlations and impulse-response

[2]For further discussion of causality concepts, the reader may consult Aigner and Zellner (1988), Bunge (1979), Eells (1991), Pearl (2000), Salmon (1984), Spirtes, Glymour, and Scheines (1993), and Sosa and Tooley (1993).

[3]For further discussion of this issue, see McCloskey and Ziliak (1996).

functions at horizons greater than one is due to indirect causal effects, as shown in Dufour and Renault (1998).[4]

In studying the relationship between volatility and returns, *implied volatility*—derived from option prices—can be an interesting alternative measure of volatility or constitutes a useful auxiliary variable because option prices may capture anticipative additional relevant information as well as nonlinear relations. Thus, implied volatility can be viewed as a forward-looking measure of volatility with an horizon corresponding to the maturity of the option. We find that adding implied volatility to the information set leads to statistical evidence for a sizable volatility feedback effect for a few days, whereas the leverage effect remains almost the same. A key element of the volatility feedback mechanism is an increase of expected future volatility. Implied volatility certainly provides an option market forecast of future volatility, which is better than a forecast based on past realized volatility.[5]

This finding can be contrasted with the one of Masset and Wallmeier (2010), who also used high-frequency data to analyze the lead–lag relationship of option implied volatility and index return in Germany, using Granger causality tests (at horizon one) and impulse-response functions. They find that the relationship is return-driven in the sense that index returns Granger cause volatility changes. Instead, through a more complete analysis based on the concept of causality at different horizons, we find that *implied volatilities* are important for assessing the *volatility feedback effect*. Our result is, however, broadly consistent with Bollerslev and Zhou (2005), who provide a model-based rationalization for finding such evidence on the volatility feedback effect through implied volatility. Based on a stochastic volatility model, they show that the relation between returns and implied volatility remains positive for all reasonable configurations of parameters.

Another contribution of this paper consists in showing that the proposed causality measures help to quantify the dynamic impact of bad and good return news on volatility.[6] A common approach to visualize the relationship between news and volatility is provided by the news-impact curve originally studied by Pagan and Schwert (1990) and Engle and Ng (1993). To study the effect of current return shocks on future expected volatility, Engle and Ng (1993) introduced the news impact function (hereafter NIF). The basic idea of this function is to consider

[4] Bollerslev, Kretschmer et al. (2009) further decompose realized volatility into two components, the continuous-path measure of volatility and the discontinuous jump component. Their results suggest that the leverage effect works primarily through the continuous volatility component.

[5] The informational content of implied volatility does not come as a surprise since several studies have documented that implied volatility can be used to predict whether a market is likely to move higher or lower and help to predict future volatility (see Day and Lewis 1992; Canina and Figlewski 1993, Lamoureux and Lastrapes 1993; Fleming 1998; Poteshman 2000; Blair, Poon, and Taylor 2001; Busch; Christensen, and Nielsen 2011). Pooling the information contained in futures and options markets unveils an effect that cannot be found with one market alone.

[6] In this study, bad and good news are determined by negative and positive innovations in returns and volatility. The impact of macroeconomic news announcements on financial markets (e.g., volatility) has also been studied by several authors (see Schwert 1981; Pearce and Roley 1985; Hardouvelis 1987; Jain 1988; Haugen, Talmor, and Torous 1991; McQueen and Roley 1993; Balduzzi, Elton, and Green 2001; Andersen, Bollerslev, Diebold and vega 2003; Huang 2007).

the effect of the return shock at time t on volatility at time $t+1$, while conditioning on information available at time t and earlier. Engle and Ng (1993) explain that this curve, where all the lagged conditional variances are evaluated at the level of the asset return unconditional variance, relates past positive and negative returns to current volatility.

We propose a new curve, the *causal news impact function (CNIF)*, for capturing the impact of news on volatility based on causality measures. In contrast with the NIF of Engle and Ng (1993), the CNIF curve can be constructed for parametric and stochastic volatility models, and it allows one to consider all the past information about volatility and returns. We also build confidence intervals using a bootstrap technique around the CNIF curve. Further, we can visualize the impact of news on volatility at different horizons (see also Chen and Ghysels 2011) rather than only one horizon as in Engle and Ng (1993).

We confirm by simulation that the CNIF based on causality measures detects well the differential effect of good and bad news in various parametric volatility models. Then, we apply the concept to the S&P 500 Index futures returns and volatility: we find a much stronger impact from bad news at several horizons. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons.

Our results on the informational value of implied volatility also suggest that the difference between implied and realized volatility (called the *variance risk premium*) constitutes an interesting measure of "news" coming to the market. So we compute causality measures from positive and negative variance risk premiums to returns. We find a stronger impact when the difference is positive (an anticipated increase in volatility or bad news) than when it is negative.

Clearly, none of the earlier studies on the relationship between returns and volatility has exploited the new methodology proposed in this paper. But our results nicely complement those of Bollerslev, Tauchen, and Zhou (2009b) and Bollerslev, Sizova, and Tauchen (2009a). Using high-frequency intraday returns on the S&P500 index and the volatility index (VIX), Bollerslev, Tauchen, and Zhou (2009b) show that the variance risk premium is able to explain a nontrivial fraction of the time-series variation in post-1990 aggregate stock market returns, with high (low) premiums predicting high (low) future returns, at a quarterly frequency. They also observe that it is consistent with the predictions of a long-run volatility risk equilibrium model. Bollerslev, Sizova, and Tauchen (2009a) rely on an equilibrium continuous-time model to capture this fact as well as the asymmetry in the relationship between volatility and past and future returns (leverage and volatility feedback effects).

Other empirical studies on the link between returns and volatility are based on lower frequency data or model-based measures of volatility (see Christie 1982; French, Schwert, and Stambaugh 1987; Schwert 1989; Turner, Startz, and Nelson 1989; Nelson 1991; Glosten, Jagannathan, and Runkle 1993; Campbell and Hentschel 1992; Bekaert and Wu 2000; Whaley 2000; Ghysels, Santa-Clara, and Valkanov 2004; Giot 2005; Ludvigson and Ng 2005; Dennis, Mayhew, and Stivers

2006; Guo and Savickas 2006, among others). On the relationship and the relative importance of the leverage and volatility feedback effects, the results of this literature are often ambiguous, if not contradictory. In particular, studies focusing on the leverage hypothesis conclude that the latter cannot completely account for changes in volatility (see Christie 1982; Schwert 1989). However, for the volatility feedback effect, empirical findings conflict. French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and Ghysels, Santa-Clara, and Valkanov (2004) find a positive relation between volatility and expected returns, while Turner, Startz, and Nelson (1989), Glosten, Jagannathan, and Runkle (1993), and Nelson (1991) find a negative relation. From individual-firm data, Bekaert and Wu (2000) conclude that the volatility feedback effect dominates the leverage effect empirically. The coefficient linking volatility to returns is often not statistically significant. Ludvigson and Ng (2005) find a strong positive contemporaneous relation between the conditional mean and conditional volatility and a strong negative lag-volatility-in-mean effect. Guo and Savickas (2006) conclude that the stock market risk–return relation is positive, as stipulated by the Capital Assest Pricing Model; however, idiosyncratic volatility is negatively related to future stock market returns. Giot (2005) and Dennis, Mayhew, and Stivers (2006) use lower frequency data (such as, daily data) to study the relationship between returns and implied volatility. Giot (2005) uses the S&P 100 index and an implied VIX to show that there is a *contemporaneous* asymmetric relationship between S&P 100 index returns and VIX: negative S&P 100 index returns yield bigger changes in VIX than do positive returns (see Whaley 2000). Dennis, Mayhew, and Stivers (2006), using daily stock returns and innovations in option-derived implied volatilities, show that the relation between stock returns and innovations in systematic volatility (idiosyncratic volatility) is substantially negative (near zero).

The plan of the paper is as follows. In Section 1, we define volatility measures in high-frequency data and we review the concept of causality at different horizons and its measures. In Section 2, we propose and discuss VAR models that allow us to measure leverage and volatility feedback effects with high-frequency data. In Section 3, we introduce information implied volatility (IV)—in addition to realized volatility and returns—to measure the dynamic leverage and volatility feedback effects. Section 4 describes the high-frequency data, the estimation procedure, and the empirical findings regarding causality effects between volatility and returns. In Section 5, we propose a method to assess the dynamic impact of good and bad return news on volatility. Simulation results on the efficiency of this method are also presented. Our empirical results on news effects in S&P 500 futures market appear in Section 6. We conclude in Section 7.

1 VOLATILITY AND CAUSALITY MEASURES

To assess causality between volatility and returns at high frequency, we need to build measures for both volatility and causality. For volatility, we use various measures of realized volatility introduced by Andersen et al. (2010) (see also Andersen and Bollerslev 1998; Andersen et al. 2001b; Barndorff-Nielsen and Shephard 2002a; Barndorff-Nielsen and Shephard 2002b). For causality, we rely on the short- and long-run causality measures proposed by Dufour and Taamouti (2010). Let us first set some notations. We denote by p_t the logarithmic price of the risky asset or portfolio (at time t) and by $r_{t+1} = p_{t+1} - p_t$ the continuously compounded return from time t to $t+1$. We assume that the price process may exhibit both stochastic volatility and jumps. It could belong to the class of continuous-time jump diffusion processes,

$$\mathrm{d}p_t = \mu_t \,\mathrm{d}t + \sigma_t \,\mathrm{d}W_t + \kappa_t \,\mathrm{d}q_t, \quad 0 \leqslant t \leqslant T, \tag{1.1}$$

where μ_t is a continuous and locally bounded variation process, σ_t is the stochastic volatility process, W_t denotes a standard Brownian motion, and $\mathrm{d}q_t$ is a counting process such that $\mathrm{d}q_t = 1$ represents a jump at time t (and $\mathrm{d}q_t = 0$ no jump) with jump intensity λ_t. The parameter κ_t refers to the size of the corresponding jumps. Thus, the quadratic variation of returns from time t to $t+1$ is given by

$$[r, r]_{t+1} = \int_t^{t+1} \sigma_s^2 \,\mathrm{d}s + \sum_{0<s\leqslant t} \kappa_s^2,$$

where the first component, called integrated volatility, comes from the continuous component of (1.1) and the second term is the contribution from discrete jumps. In the absence of jumps, the second term on the right-hand side disappears and the quadratic variation is simply equal to the integrated volatility.

1.1 Volatility in High-Frequency Data: Realized Volatility, Bipower Variation, Jumps

In this section, we define the high-frequency measures that we shall use to capture volatility. In what follows, we normalize the daily time interval to unity and we divide it into h periods. Each period has length $\Delta = 1/h$. Let the discretely sampled Δ-period returns be denoted by $r_{(t,\Delta)} = p_t - p_{t-\Delta}$ and the daily return by $r_{t+1} = \sum_{j=1}^{h} r_{(t+j\Delta,\Delta)}$. The daily realized volatility is defined as the summation of the corresponding h high-frequency intradaily squared returns:

$$\mathrm{RV}_{t+1} \equiv \sum_{j=1}^{h} r^2_{(t+j\Delta,\Delta)}.$$

The realized volatility satisfies

$$\lim_{\Delta\to 0} \mathrm{RV}_{t+1} = \int_t^{t+1} \sigma_s^2 \,\mathrm{d}s + \sum_{0<s\leqslant t} \kappa_s^2, \tag{1.2}$$

which means that RV_{t+1} is a consistent estimator of the sum of the integrated variance $\int_t^{t+1} \sigma_s^2 \,\mathrm{d}s$ and the jump contribution (see Andersen and Bollerslev 1998, Andersen et al. 2001b, 2010; Barndorff-Nielsen and Shephard 2002a, Barndorff-Nielsen and Shephard 2002b; Comte and Renault 1998).[7] Similarly, a measure of standardized bipower variation is given by

$$\mathrm{BV}_{t+1} \equiv \frac{\pi}{2} \sum_{j=2}^{h} |r_{(t+j\Delta,\Delta)}||r_{(t+(j-1)\Delta,\Delta)}|.$$

Under reasonable assumptions on the dynamics of (1.1), the bipower variation satisfies

$$\lim_{\Delta \longrightarrow 0} \mathrm{BV}_{t+1} = \int_t^{t+1} \sigma_s^2 \, \mathrm{d}s \tag{1.3}$$

(see Barndorff-Nielsen and Shephard 2004; Barndorff-Nielsen et al. 2005). Equation (1.3) means that BV_{t+1} provides a consistent estimator of the integrated variance unaffected by jumps. Finally, as noted by Barndorff-Nielsen and Shephard (2004), combining the results in Equations (1.2) and (1.3), the contribution to the quadratic variation due to discontinuities (jumps) in the underlying price process may be consistently estimated by

$$\lim_{\Delta \longrightarrow 0} (\mathrm{RV}_{t+1} - \mathrm{BV}_{t+1}) = \sum_{0<s\leqslant t} \kappa_s^2. \tag{1.4}$$

We can also define the relative measure

$$\mathrm{RJ}_{t+1} = \frac{(\mathrm{RV}_{t+1} - \mathrm{BV}_{t+1})}{\mathrm{RV}_{t+1}} \tag{1.5}$$

or the corresponding logarithmic ratio

$$\bar{J}_{t+1} = \ln(\mathrm{RV}_{t+1}) - \ln(\mathrm{BV}_{t+1}).$$

Huang and Tauchen (2005) argue that these are more robust measures of the contribution of jumps to total price variation. Since in practice $\bar{J}_{t+1}$ can be negative in a given sample, we impose a nonnegativity truncation of the actual empirical jump measurements:

$$J_{t+1} \equiv \max[\ln(\mathrm{RV}_{t+1}) - \ln(\mathrm{BV}_{t+1}), 0]$$

(see Barndorff-Nielsen and Shephard 2004; Andersen et al. 2010).

1.2 Short-Run and Long-Run Causality Measures

We study causality at different horizons between returns (r_t) and volatilities (σ_t^2). For that purpose, it will be convenient to define first *noncausality* in terms of orthogonality between subspaces of a Hilbert space of random variables with finite

[7]For a general discussion of integrated and realized volatilities in the absence of jumps, see Meddahi (2002).

second moments. To give a formal definition of noncausality at different horizons, we need to consider the following notations. We denote by $r(\omega, t]$, $\sigma^2(\omega, t]$, and $z(\omega, t]$ the information contained in the history of variables of interest r and σ^2 and another auxiliary variable z, respectively, up to time t. The "starting point" ω is typically equal to a finite initial date (such as $\omega = -1, 0$, or 1) or to $-\infty$. In our empirical application, the auxiliary variable z is given by the implied volatility (hereafter IV). The information sets obtained by "adding" $z(\omega, t]$ to $r(\omega, t]$, $z(\omega, t]$ to $\sigma^2(\omega, t]$, $r(\omega, t]$ to $\sigma^2(\omega, t]$, and $z(\omega, t]$ to $r(\omega, t]$ and $\sigma^2(\omega, t]$ are defined as follows:

$$I_{rz}(t) = I_0 + r(\omega, t] + z(\omega, t], \quad I_{\sigma^2 z}(t) = I_0 + \sigma^2(\omega, t] + z(\omega, t],$$

$$I_{r\sigma^2}(t) = I_0 + r(\omega, t] + \sigma^2(\omega, t], \quad I_{r\sigma^2 z}(t) = I_0 + r(\omega, t] + \sigma^2(\omega, t] + z(\omega, t],$$

where I_0 represents a fundamental information set available in all cases (such as deterministic variables, a constant, etc.). Finally, for any given information set B_t (some Hilbert subspace) and positive integer h, we denote by $P[r_{t+h}|B_t]$ (respectively $P[\sigma^2_{t+h}|B_t]$) the best linear forecast of r_{t+h} (respectively σ^2_{t+h}) based on the information set B_t and $u[r_{t+h}|B_t] = r_{t+h} - P[r_{t+h}|B_t]$ (respectively $u[\sigma^2_{t+h}|B_t] = \sigma^2_{t+h} - P[\sigma^2_{t+h}|B_t]$) the corresponding prediction error.[8] Thus, we have the following definition of noncausality at different horizons (see Dufour and Renault 1998; Dufour and Taamouti 2010).

Definition 1.1. *Let* h be a positive integer.
(i) r does not cause σ^2 at horizon h given $I_{\sigma^2 z}(t)$, denoted $r \underset{h}{\nrightarrow} \sigma^2 | I_{\sigma^2 z}(t)$, iff

$$\mathrm{Var}[u[\sigma^2_{t+h}|I_{\sigma^2 z}(t)]] = \mathrm{Var}[u[\sigma^2_{t+h}|I_{r\sigma^2 z}(t)]] \ ;$$

(ii) r does not cause σ^2 up to horizon h given $I_{\sigma^2 z}(t)$, denoted $r \underset{(h)}{\nrightarrow} \sigma^2 | I_{\sigma^2 z}(t)$, iff

$$r \underset{k}{\nrightarrow} \sigma^2 | I_{\sigma^2 z}(t) \quad \text{for } k = 1, 2, \ldots, h \ ;$$

(iii) r does not cause σ^2 at any horizon given $I_{\sigma^2 z}(t)$, denoted $r \underset{(\infty)}{\nrightarrow} \sigma^2 | I_{\sigma^2 z}(t)$, iff

$$r \underset{k}{\nrightarrow} \sigma^2 | I_{\sigma^2 z}(t) \quad \text{for all } k = 1, 2, \ldots .$$

Definition 1.1 corresponds to causality from r to σ^2 and means that r causes σ^2 at horizon h if the past of r improves the forecast of σ^2_{t+h} given the information set $I_{\sigma^2 z}(t)$. We can similarly define noncausality at horizon h from σ^2 to r. The presence of the auxiliary variable z may transmit causality between r and σ^2 at horizon h

[8] B_t can be equal to $I_{r\sigma^2 z}(t)$, $I_{rz}(t)$, or $I_{\sigma^2 z}(t)$.

strictly higher than one, even if there is no causality between the two variables at horizon one. However, in the absence of auxiliary variable, noncausality at horizon one implies noncausality at any horizon h strictly higher than one (see Dufour and Renault 1998). In other words,

$$r \underset{1}{\nrightarrow} \sigma^2 | I_{\sigma^2}(t) \Rightarrow r \underset{(\infty)}{\nrightarrow} \sigma^2 | I_{\sigma^2}(t),$$

$$\sigma^2 \underset{1}{\nrightarrow} r | I_r(t) \Rightarrow \sigma^2 \underset{(\infty)}{\nrightarrow} r | I_r(t),$$

where $I_{\sigma^2}(t) = I_0 + \sigma^2(\omega, t]$ and $I_r(t) = I_0 + r(\omega, t]$. A measure of causality from r to σ^2 at horizon h, denoted $C(r \underset{h}{\longrightarrow} \sigma^2)$, is given by the following function (see Dufour and Taamouti (2010)):

$$C\left(r \underset{h}{\longrightarrow} \sigma^2\right) = \ln\left[\frac{\mathsf{Var}[u[\sigma^2_{t+h}|I_{\sigma^2 z}(t)]]}{\mathsf{Var}[u[\sigma^2_{t+h}|I_{r\sigma^2 z}(t)]]}\right].$$

Similarly, a measure of causality from σ^2 to r at horizon h, denoted $C(\sigma^2 \underset{h}{\longrightarrow} r)$, is given by

$$C\left(\sigma^2 \underset{h}{\longrightarrow} r\right) = \ln\left[\frac{\mathsf{Var}[u[r_{t+h}|I_{rz}(t)]]}{\mathsf{Var}[u[r_{t+h}|I_{r\sigma^2 z}(t)]]}\right].$$

For example, $C(r \underset{h}{\longrightarrow} \sigma^2)$ measures the causal effect from r to σ^2 at horizon h given the past of σ^2 and z. In terms of predictability, it measures the information given by the past of r that can improve the forecast of σ^2_{t+h}. Since $\mathsf{Var}[u[\sigma^2_{t+h}|I_{\sigma^2 z}(t)]] \geqslant \mathsf{Var}[u[\sigma^2_{t+h}|I_{r\sigma^2 z}(t)]]$, the function $C(r \underset{h}{\longrightarrow} \sigma^2)$ is nonnegative. Furthermore, it is zero when there is no causality at horizon h. However, as soon as there is causality at horizon one, causality measures at different horizons may considerably differ.

In Dufour and Taamouti (2010), a measure of instantaneous causality between r and σ^2 at horizon h is also proposed. It is given by the function

$$C\left(r \underset{h}{\leftrightarrow} \sigma^2\right) = \ln\left[\frac{\mathsf{Var}[u[r_{t+h}|I_{r\sigma^2 z}(t)]]\mathsf{Var}[u[\sigma^2_{t+h}|I_{r\sigma^2 z}(t)]]}{\det(\Sigma[r_{t+h}, \sigma^2_{t+h}|I_{r\sigma^2 z}(t)])}\right],$$

where $\det(\Sigma[r_{t+h}, \sigma^2_{t+h}|I_{r\sigma^2 z}(t)])$ represents the determinant of the variance–covariance matrix $\Sigma[r_{t+h}, \sigma^2_{t+h}|I_{r\sigma^2 z}(t)]$ of the forecast error of the joint process $(r, \sigma^2)'$ at horizon h given the information set $I_{r\sigma^2 z}(t)$. Note that σ^2 may be replaced by $\ln(\sigma^2)$. Since the logarithmic transformation is nonlinear, this may modify the value of the causality measure.

In what follows, we apply the above measures to study causality at different horizons from returns to volatility (hereafter leverage effect), from volatility to returns (hereafter volatility feedback effect), and the instantaneous causality and dependence between returns and volatility. In Section 2, we study these effects by considering a limited information set which only contains the past of returns and realized volatility. In Section 3, we include lagged implied volatility in the information set.

2 MEASURING LEVERAGE AND VOLATILITY FEEDBACK EFFECTS IN A VAR MODEL

In this section, we study the relationship between the return r_t and its volatility σ_t^2. The objective is to measure and compare the strength of *dynamic* leverage and volatility feedback effects in high-frequency equity data. These effects are quantified within the context of a VAR model and by using short- and long-run causality measures proposed by Dufour and Taamouti (2010). The asymmetric volatility phenomenon can be the result of causality from returns to volatility (leverage effect), from volatility to returns (volatility feedback effect), or instantaneous causality. We wish to measure and compare these effects in order to determine the most important ones.

We suppose that the joint process of returns and logarithmic volatility, $(r_{t+1}, \ln(\sigma_{t+1}^2))'$, follows an autoregressive linear model

$$\begin{pmatrix} r_{t+1} \\ \ln(\sigma_{t+1}^2) \end{pmatrix} = \begin{pmatrix} \mu_r \\ \mu_\sigma \end{pmatrix} + \sum_{j=1}^{p} \begin{bmatrix} \Phi_{11j} & \Phi_{12j} \\ \Phi_{21j} & \Phi_{22j} \end{bmatrix} \begin{pmatrix} r_{t+1-j} \\ \ln(\sigma_{t+1-j}^2) \end{pmatrix} + \begin{pmatrix} u_{t+1}^r \\ u_{t+1}^\sigma \end{pmatrix}, \tag{2.1}$$

with $\mathsf{E}[u_t] = 0$ and $\mathsf{Var}[u_t] = \Sigma_u$, where $u_t = (u_t^r, u_t^\sigma)'$. In the empirical application, σ_{t+1}^2 will be replaced by the realized volatility RV_{t+1} or the bipower variation BV_{t+1}. The disturbance u_{t+1}^r is the one-step-ahead error when r_{t+1} is forecast from its own past and the past of $\ln(\sigma_{t+1}^2)$, and similarly u_{t+1}^σ is the one-step-ahead error when $\ln(\sigma_{t+1}^2)$ is forecast from its own past and the past of r_{t+1}. We suppose that these disturbances are each serially uncorrelated but may be correlated with each other contemporaneously and at various leads and lags. Since u_{t+1}^r is uncorrelated with $I_{r\sigma^2}(t)$, the equation for r_{t+1} represents the linear projection of r_{t+1} on $I_{r\sigma^2}(t)$. Likewise, the equation for $\ln(\sigma_{t+1}^2)$ represents the linear projection of $\ln(\sigma_{t+1}^2)$ on $I_{r\sigma^2}(t)$.

Equation (2.1) models the first two conditional moments of the asset returns. We represent conditional volatility as an exponential function process to guarantee that it is positive. The first equation in Equation (2.1) describes the dynamics of the return as

$$r_{t+1} = \mu_r + \sum_{j=1}^{p} \Phi_{11j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{12j} \ln(\sigma_{t+1-j}^2) + u_{t+1}^r.$$

This equation allows to capture the temporary component of Fama and French (1988) permanent and temporary components model, in which stock prices are governed by a random walk and a stationary autoregressive process, respectively.

For $\Phi_{12j} = 0$, this model of the temporary component is the same as that of Lamoureux and Lastrapes (1993) (see also Brandt and Kang 2004; Whitelaw 1994). The second equation in Equation (2.1) describes the volatility dynamics as

$$\ln(\sigma^2_{t+1}) = \mu_\sigma + \sum_{j=1}^{p} \Phi_{21j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{22j} \ln(\sigma^2_{t+1-j}) + u^\sigma_{t+1}, \tag{2.2}$$

which is a stochastic volatility model. For $\Phi_{21j} = 0$, Equation (2.2) can be viewed as the stochastic volatility model estimated by Wiggins (1987), Andersen and Sørensen (1996), and many others. However, in this paper, we consider that σ^2_{t+1} is not a latent variable and it can be approximated by realized or bipower variations from high-frequency data. We also note that the conditional mean equation includes the volatility-in-mean model used by French, Schwert, and Stambaugh (1987) and Glosten, Jagannathan, and Runkle (1993) to explore the contemporaneous relationship between the conditional mean and volatility (see Brandt and Kang 2004). To illustrate the connection to the volatility-in-mean model, we premultiply the system in Equation (2.1) by the matrix

$$P = \begin{bmatrix} 1 & -\frac{\mathrm{Cov}(r_{t+1}, \ln(\sigma^2_{t+1}))}{\mathrm{Var}[\ln(\sigma^2_{t+1}) | I_{r\sigma^2}(t)]} \\ -\frac{\mathrm{Cov}(r_{t+1}, \ln(\sigma^2_{t+1}))}{\mathrm{Var}[r_{t+1} | I_{r\sigma^2}(t)]} & 1 \end{bmatrix}.$$

Then, the first equation of r_{t+1} is a linear function of the elements of $r(\omega, t]$, $\sigma^2(\omega, t+1]$, and the disturbance $u^r_{t+1} - \frac{\mathrm{Cov}(r_{t+1}, \ln(\sigma^2_{t+1}))}{\mathrm{Var}[\ln(\sigma^2_{t+1}) | I_{r\sigma^2}(t)]} u^\sigma_{t+1}$. Since this disturbance is uncorrelated with u^σ_{t+1}, it is uncorrelated with $\ln(\sigma^2_{t+1})$ as well as with $r(\omega, t]$ and $\sigma^2(\omega, t+1]$. Hence, the linear projection of r_{t+1} on $r(\omega, t]$ and $\sigma^2(\omega, t+1]$ is provided by the first equation of the new system:

$$r_{t+1} = \nu_r + \sum_{j=1}^{p} \phi_{11j} r_{t+1-j} + \sum_{j=0}^{p} \phi_{12j} \ln(\sigma^2_{t+1-j}) + \tilde{u}^r_{t+1}. \tag{2.3}$$

The new parameters ν_r, ϕ_{11j}, and ϕ_{12j}, for $j = 0, 1, \ldots, p$, are functions of parameters in the vector μ and matrix Φ_j, for $j = 1, \ldots, p$. Equation (2.3) is a generalized version of the usual volatility-in-mean model, in which the conditional mean depends contemporaneously on the conditional volatility. Similarly, the existence of the linear projection of $\ln(\sigma^2_{t+1})$ on $r(\omega, t+1]$ and $\sigma^2(\omega, t]$,

$$\ln(\sigma^2_{t+1}) = \nu_\sigma + \sum_{j=0}^{p} \phi_{21j} r_{t+1-j} + \sum_{j=1}^{p} \phi_{22j} \ln(\sigma^2_{t+1-j}) + \tilde{u}^\sigma_{t+1}, \tag{2.4}$$

follows from the second equation of the new system. The new parameters ν_σ, ϕ_{21j}, and ϕ_{22j}, for $j = 0, 1, \ldots, p$, are functions of parameters in the vector μ and matrices Φ_j, $j = 1, \ldots, p$. The volatility model given by Equation (2.4) captures the persistence of volatility through the coefficients ϕ_{22j}. In addition, it incorporates the effects of the mean on volatility, both at the contemporaneous and at the intertemporal levels through the coefficients ϕ_{21j}, for $j = 0, 1, \ldots, p$.

Based on system (2.1), the forecast error of $(r_{t+h}, \ln(\sigma^2_{t+h}))'$ is given by

$$e[(r_{t+h}, \ln(\sigma^2_{t+h}))'] = \sum_{i=0}^{h-1} \psi_i u_{t+h-i}, \tag{2.5}$$

where the coefficients ψ_i are the impulse-response coefficients of the MA(∞) representation of (2.1). The covariance matrix of the forecast error (2.5) is given by

$$\mathsf{Var}[e[(r_{t+h}, \ln(\sigma^2_{t+h}))']] = \sum_{i=0}^{h-1} \psi_i \Sigma_u \psi_i'. \tag{2.6}$$

We also consider the following restricted model:

$$\begin{pmatrix} r_{t+1} \\ \ln(\sigma^2_{t+1}) \end{pmatrix} = \begin{pmatrix} \bar{\mu}_r \\ \bar{\mu}_\sigma \end{pmatrix} + \sum_{j=1}^{\bar{p}} \begin{bmatrix} \bar{\Phi}_{11j} & 0 \\ 0 & \bar{\Phi}_{22j} \end{bmatrix} \begin{pmatrix} r_{t+1-j} \\ \ln(\sigma^2_{t+1-j}) \end{pmatrix} + \begin{pmatrix} \bar{u}^r_{t+1} \\ \bar{u}^\sigma_{t+1} \end{pmatrix} \tag{2.7}$$

with $\mathsf{E}[\bar{u}_t] = 0$ and $\mathsf{Var}[\bar{u}_t] = \bar{\Sigma}_u$, where $\bar{u}_t = (\bar{u}^r_t, \bar{u}^\sigma_t)'$. Zero values in the autoregressive matrix coefficients mean that there is noncausality at horizon one from returns to volatility and from volatility to returns. As mentioned in Section 1.2, in a bivariate system, noncausality at horizon one implies noncausality at any horizon h strictly higher than one. This means that the absence of leverage effect at horizon one (respectively the absence of volatility feedback effect at horizon one), which corresponds to $\bar{\Phi}_{21j} = 0$, for $j = 1, \ldots, \bar{p}$ (respectively $\bar{\Phi}_{12j} = 0$, for $j = 1, \ldots, \bar{p}$), is equivalent to the absence of leverage effect (respectively volatility feedback effect) at any horizon $h \geqslant 1$.

To compare the forecast error variance of model (2.1) with that of model (2.7), we assume that $p = \bar{p}$. Based on the restricted model (2.7), the covariance matrix of the forecast error of $(r_{t+h}, \ln(\sigma^2_{t+h}))'$ is given by

$$\overline{\mathsf{Var}}[\bar{e}[(r_{t+h}, \ln(\sigma^2_{t+h}))']] = \sum_{i=0}^{h-1} \bar{\psi}_i \bar{\Sigma}_{\bar{u}} \bar{\psi}_i', \tag{2.8}$$

where the coefficients $\bar{\psi}_i$, for $i = 0, \ldots, h-1$, represent the impulse-response coefficients of the MA(∞) representation of model (2.7). From the covariance matrices (2.6) and (2.8), we define the following measures of leverage and volatility feedback effects at any horizon h, where $h \geqslant 1$,

$$\mathrm{C}\left(r \underset{h}{\longrightarrow} \ln(\sigma^2)\right) = \ln\left[\frac{\sum_{i=0}^{h-1} e_2'(\bar{\psi}_i \bar{\Sigma}_{\bar{u}} \bar{\psi}_i')e_2}{\sum_{i=0}^{h-1} e_2'(\psi_i \Sigma_u \psi_i')e_2}\right], \quad e_2 = (0, 1)', \tag{2.9}$$

$$C\left(\ln(\sigma^2) \underset{h}{\longrightarrow} r\right) = \ln\left[\frac{\sum_{i=0}^{h-1} e_1'(\bar{\psi}_i \bar{\Sigma}_{\bar{u}} \bar{\psi}_i')e_1}{\sum_{i=0}^{h-1} e_1'(\psi_i \Sigma_u \psi_i')e_1}\right], \quad e_1 = (1,0)'. \tag{2.10}$$

The parametric measure of instantaneous causality at horizon h, where $h \geqslant 1$, is given by the following function:

$$C\left(r \underset{h}{\leftrightarrow} \ln(\sigma^2)\right) = \ln\left[\frac{\left(\sum_{i=0}^{h-1} e_2'(\psi_i \Sigma_u \psi_i')e_2\right)\left(\sum_{i=0}^{h-1} e_1'(\psi_i \Sigma_u \psi_i')e_1\right)}{\det\left(\sum_{i=0}^{h-1} \psi_i \Sigma_u \psi_i'\right)}\right].$$

3 IMPLIED VOLATILITY AS AN AUXILIARY VARIABLE

An important feature of causality is the information set considered to forecast the variables of interest. Until now, we have included only the past of returns and realized volatility. Since the volatility feedback effect rests on anticipating future movements in volatility, it is natural to include option-based implied volatility, an all-important measure of market expectations of future volatility. Formally, we "add" the past of implied volatility to the information set $I_{r\sigma^2}(t)$ considered in the previous section. The new information set is given now by $I_{r\sigma^2 z}(t)$, where z is an auxiliary variable represented by implied volatility.

To take implicit volatility into account, we consider call options written on S&P 500 index futures contracts. The data come from the OptionMetrics dataset on option prices, dating back to January 1996. Given observations on the option price C and the remaining variables S, K, τ, and r, an estimate of the implied volatility IV can be obtained by solving the nonlinear equation $C = C(S, K, \tau, r, \mathrm{IV}^{1/2})$ for $\mathrm{IV}^{1/2}$, where $C(\cdot)$ refers to the Black–Scholes formula. Each day, we extract the implied volatility corresponding to the option that is closest to the money. This selection criterion ensures that the option will be liquid and therefore aggregates the opinion of many investors about future volatility. This appears more important than keeping a fixed maturity. This choice is often made in the empirical literature on option pricing (see, e.g., Pan 2002). Summary statistics for the daily implied volatility ($\mathrm{IV}^{1/2}$), squared implied volatility (IV) and logarithm of squared implied volatility ($\ln(\mathrm{IV})$) are reported in Table 3.

Therefore, we consider a trivariate autoregressive model including implied volatility, in addition to the realized volatility (bipower variation) and returns:[9]

$$\begin{bmatrix} r_{t+1} \\ \mathrm{RV}^*_{t+1} \\ \mathrm{IV}^*_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_r \\ \mu_{\mathrm{RV}} \\ \mu_{\mathrm{IV}} \end{bmatrix} + \sum_{j=1}^{p} \begin{bmatrix} \Phi_{11j} & \Phi_{12j} & \Phi_{13j} \\ \Phi_{21j} & \Phi_{22j} & \Phi_{23j} \\ \Phi_{31j} & \Phi_{32j} & \Phi_{33j} \end{bmatrix} \begin{bmatrix} r_{t+1-j} \\ \mathrm{RV}^*_{t+1-j} \\ \mathrm{IV}^*_{t+1-j} \end{bmatrix} + \begin{bmatrix} u^r_{t+1} \\ u^{\mathrm{RV}}_{t+1} \\ u^{\mathrm{IV}}_{t+1} \end{bmatrix}, \tag{3.1}$$

where $\mathrm{RV}^*_t = \ln(\mathrm{RV}_t)$ and $\mathrm{IV}^*_t = \ln(\mathrm{IV}_t)$. The first equation of the above system

$$r_{t+1} = \mu_r + \sum_{j=1}^{p} \Phi_{11j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{12j} \mathrm{RV}^*_{t+1-j} + \sum_{j=1}^{p} \Phi_{13j} \mathrm{IV}^*_{t+1-j} + u^r_{t+1} \tag{3.2}$$

[9]Further, we consider an autoregressive model where we add jumps and our results do not change.

describes the dynamics of the return, while the second equation

$$RV^*_{t+1} = \mu_{\mathrm{RV}} + \sum_{j=1}^{p} \Phi_{21j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{22j} \mathrm{RV}^*_{t+1-j} + \sum_{j=1}^{p} \Phi_{23j} \mathrm{IV}^*_{t+1-j} + u^{\mathrm{RV}}_{t+1} \quad (3.3)$$

describes the volatility dynamics. It is well known that implied volatility can be used to predict whether a market is likely to move higher or lower and help to predict future volatility (see Day and Lewis 1992; Canina and Figlewski 1993; Lamoureux and Lastrapes 1993; Poteshman 2000; Blair, Poon, and Taylor 2001; Busch; Christensen, and Nielsen 2011). The forward-looking nature of the implied volatility measure makes it an ideal additional variable to capture a potential volatility feedback mechanism. Apart from using IV without any constraint in (3.2) and (3.3), we will also look at more restricted combinations dictated by financial considerations. Indeed, the difference between IV and RV provides an estimate of the risk premium attributable to the variance risk factor.

4 CAUSALITY MEASURES FOR S&P 500 FUTURES

In this section, we first describe the data used to measure causality in the VAR models of the previous sections. Then we explain how to estimate confidence intervals of causality measures for leverage and volatility feedback effects. Finally, we discuss our findings.

4.1 Data Description

Our data consist of high-frequency tick-by-tick transaction prices on S&P 500 Index futures contracts traded on the Chicago Mercantile Exchange, over the period January 1988 to December 2005 for a total of 4494 trading days. We eliminated a few days where trading was thin and the market was open for a shortened session. Due to the unusually high volatility at the opening, we also omit the first five minutes of each trading day (see Bollerslev, Litvinova, and Tauchen 2006). For reasons associated with microstructure effects, we follow Bollerslev, Litvinova, and Tauchen (2006) and the literature in general and aggregate returns over five minute intervals. We calculate the continuously compounded returns over each five minute interval by taking the difference between the logarithm of the two tick prices immediately preceding each five minute mark to obtain a total of 77 observations per day (see Müller et al. 1997; Bollerslev, Litvinova, and Tauchen 2006, for more details). We also construct hourly and daily returns by summing 11 and 77 successive five minute returns, respectively.

Summary statistics for the five minute, hourly, and daily returns and the associated volatilities are reported in Tables 1–2. From these, we see that the unconditional distributions of the returns exhibit high kurtosis and negative skewness.

The sample kurtosis is much greater than the Gaussian value of three for all series. The negative skewness remains moderate, especially for the five minute and daily returns. Similarly, the unconditional distributions of realized and bipower volatility measures are highly skewed and leptokurtic. However, on applying a logarithmic transformation, both measures appear approximately normal (see Andersen et al. 2001a). The descriptive statistics for the relative jump measure, J_{t+1}, clearly indicate a positively skewed and leptokurtic distribution.

It is also of interest to assess whether the realized and bipower volatility measures differ significantly. To test this, recall that

$$\lim_{\Delta \to 0} (\mathrm{RV}_{t+1}) = \int_t^{t+1} \sigma_s^2 \, ds + \sum_{0<s\leqslant t} \kappa_s^2,$$

where $\int_t^{t+1} \sigma_s^2 \, ds$ is the integrated volatility and $\sum_{0<s\leqslant t} \kappa_s^2$ represents the contribution of jumps to total price variation. In the absence of jumps, the second term on the right-hand side disappears, and the quadratic variation is simply equal to the

Table 1 Summary statistics for S&P 500 futures returns, 1988–2005

Variables	Mean	Standard deviation	Median	Skewness	Kurtosis
Five minute	0.0000069	0.000978	0.00000000	−0.0818	73.9998
Hourly	0.0000131	0.003100	0.00000000	−0.4559	16.6031
Daily	0.0001466	0.008900	0.00011126	−0.1628	12.3714

This table summarizes the five minute, hourly, and daily returns distributions for the S&P 500 index contracts.

Table 2 Summary statistics for daily volatilities, 1988–2005

Variables	Mean	Standard deviation	Median	Skewness	Kurtosis
Hourly					
RV_t	0.00001080	0.0000294	0.00000544	42.9510	3211.190
BV_t	0.00000932	0.0000229	0.00000455	32.1242	2023.507
$\ln(\mathrm{RV}_t)$	−12.2894	1.1475	−12.3006	0.0792	3.3157
$\ln(\mathrm{BV}_t)$	−12.1007	1.0973	−12.1217	0.1558	3.2625
J_{t+1}	0.2258	0.2912	0.1221	2.0066	8.8949
Daily					
RV_t	0.0000813	0.000120	0.0000498	8.1881	120.7530
BV_t	0.0000762	0.000109	0.0000469	6.8789	78.9491
$\ln(\mathrm{RV}_t)$	−9.8582	0.8762	−9.9076	0.4250	3.3382
$\ln(\mathrm{BV}_t)$	−9.9275	0.8839	−9.9663	0.4151	3.2841
J_{t+1}	0.0870	0.1005	0.0575	1.6630	7.3867

This table summarizes the hourly and daily volatilities distributions for the S&P 500 index contracts.

Table 3 Summary statistics for daily volatilities, 1988–2005

Variables	Mean	Standard deviation	Median	Skewness	Kurtosis
$IV_t^{1/2}$	1.1808	0.8225	1.0205	3.4518	30.5778
IV_t	2.0705	5.1356	1.0415	17.8220	484.6803
$\ln(IV_t)$	−0.0326	1.1980	0.0406	0.0676	3.0002

This table summarizes the daily implied volatilities distributions for the S&P 500 index contracts.

integrated volatility: or asymptotically ($\Delta \to 0$) the realized variance is equal to the bipower variance. Many statistics have been proposed to test for the presence of jumps in financial data (see e.g., Barndorff-Nielsen and Shephard 2002b; Andersen, Bollerslev, and Diebold 2003; Huang and Tauchen 2005). In this paper, we test for the presence of jumps in our data by considering the following test statistics:

$$z_{QP,l,t} = \frac{RV_{t+1} - BV_{t+1}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\Delta QP_{t+1}}}, \tag{4.1}$$

$$z_{QP,t} = \frac{\ln(RV_{t+1}) - \ln(BV_{t+1})}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\Delta \frac{QP_{t+1}}{BV_{t+1}^2}}}, \tag{4.2}$$

$$z_{QP,lm,t} = \frac{\ln(RV_{t+1}) - \ln(BV_{t+1})}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\Delta \max\left(1, \frac{QP_{t+1}}{BV_{t+1}^2}\right)}}, \tag{4.3}$$

where QP_{t+1} is the realized Quad–Power Quarticity (Barndorff-Nielsen and Shephard 2002a), with

$$QP_{t+1} = h\mu_1^{-4} \sum_{j=4}^{h} |r_{(t+j\cdot\Delta,\Delta)}||r_{(t+(j-1)\cdot\Delta,\Delta)}||r_{(t+(j-2)\cdot\Delta,\Delta)}||r_{(t+(j-3)\cdot\Delta,\Delta)}|$$

and $\mu_1 = \sqrt{\frac{2}{\pi}}$. Under the assumption of no jumps and for each time t, the statistics $z_{QP,l,t}$, $z_{QP,t}$, and $z_{QP,lm,t}$ follow a Normal distribution $\mathcal{N}(0,1)$ as $\Delta \to 0$. The results of testing for jumps in our data are plotted in Figure 1. These graphs represent the quantile to quantile plots (hereafter QQ plot) of the relative measure of jumps given by Equation (1.5) and the QQ Plots of the other statistics; $z_{QP,l,t}$, $z_{QP,t}$, and $z_{QP,lm,t}$. When there are no jumps, we expect that the cross line and the dotted line in Figure 1 will coincide. However, as this figure shows, the two lines are clearly distinct, indicating the presence of jumps in our data. Therefore, we will present our results for both realized volatility and bipower variation.

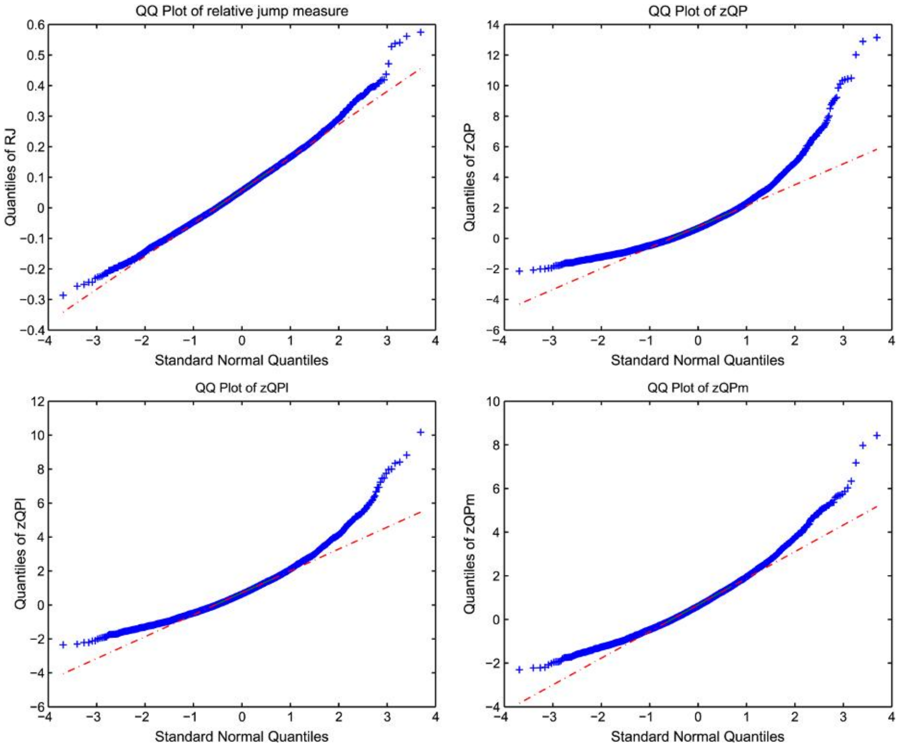

Figure 1 Quantile to quantile plots (QQ plot) of the relative measure of jumps (RJ), $z_{QP,l,t}$, $z_{QP,t}$, and $z_{QP,lm,t}$. January 1988 to December 2005.

4.2 Causality Measures

We examine several empirical issues regarding the relationship between volatility and returns. Before high-frequency data became available and the concept of realized volatility took root, such issues could only be addressed through volatility models. Recently, Bollerslev, Litvinova, and Tauchen (2006) looked at these relationships using high-frequency data and realized volatility measures. As they emphasize, the fundamental difference between the leverage and the volatility feedback explanations lies in the direction of causality. The leverage effect explains why a low return causes higher subsequent volatility, while the volatility feedback effect captures how an increase in volatility may cause a negative return. However, they studied only correlations between returns and volatility at various leads and lags, not causality relationships.

Here, we apply short-run and long-run causality measures to quantify the strength of the relationships between return and volatility at various horizons. We use ordinary least squares to estimate the VAR models described above and the Akaike information criterion to specify their orders.[10] To obtain consistent estimates of the causality measures, we simply replace the unknown parameters by their estimates. We calculate causality measures for various horizons $h = 1, \ldots, 20$. A higher value for a causality measure indicates a stronger causality. We also compute the corresponding nominal 95% bootstrap percentile confidence intervals using two different methods: the first one is based on the procedure described in Dufour and Taamouti (2010: 52) and the second one corresponds to the *fixed-design wild bootstrap* described in Gonçalves and Kilian (2004, Section 3.2). Further, the new confidence intervals were built after accounting for a possible bias in the autoregressive coefficients. [11] As mentioned by Inoue and Kilian (2002), for bounded measures, as in our case, the bootstrap approach is more reliable than the delta method. One reason is because the delta method interval is not range respecting and may produce confidence intervals that are invalid. In contrast, the bootstrap percentile interval preserves by construction these constraints (see Inoue and Kilian 2002: 315–318; Efron and Tibshirani 1993). Further, the percentile interval allows avoiding using the variance–covariance matrix of the estimators that can depend on the homoskedasticity assumption. More details on the consistency and statistical justification of the above two procedures are available in Dufour and Taamouti (2010) and Gonçalves and Kilian (2004).

The concept of Granger causality requires an information set and is analyzed in the framework of a model between the variables of interest. Both the strength of this causal link and its statistical significance are important. A major complication in detecting causality is aggregation. Low-frequency data may mask the true causal relationship between variables. High-frequency data thus offer an opportunity to analyze causal effects. In particular, we can distinguish with an exceptionally high resolution between immediate and lagged effects. Further, even if our interest focuses on relationships at the daily frequency, using higher frequency data to construct daily returns and volatilities can provide better estimates than using daily returns (as done in previous studies). Besides, since measured realized volatility can be viewed as an approximation to the "true" unobservable volatility, we consider both raw realized volatility and the bipower variation (which provides a way to filter out possible jumps in the data) (see Barndorff-Nielsen and Shephard 2004).

With five minute intervals, we could estimate the VAR model at this frequency. However, if we wish to allow for enough time for the effects to develop,

[10] Using Akaike's criterion, we find that the appropriate value of the order of the unconstrained autoregressive model is equal to 10. Since using the same criterion the value of the order of the constrained model is smaller than 10, we take $p = \bar{p} = 10$ (see Section 2).

[11] More details on bias correction in causality measures, see the end of Section 8 in Dufour and Taamouti (2010).

we need a large number of lags in the VAR model and sacrifice efficiency in the estimation. This problem arises in many studies of volatility forecasting. Researchers have used several schemes to group five minute intervals, in particular the heterogeneous autoregressive model of the realized volatility (HAR-RV) or the mixed data sampling (MIDAS) schemes. [12] We decided to look at both hourly and daily frequencies.

In this section and next ones, our empirical results will be presented mainly through graphs. Each figure reports the causality measure as a function of the horizon. To preserve space and reduce the number of graphs, we exclude almost all the graphs with confidence intervals and we focus on the main figures where different effects are compared across horizons. However, in the paper we discuss the results of statistical significance of the effects.[13]

The main results that correspond to the present section are summarized and compared in Figures 2–5. Results based on bivariate models indicate the following (see Figure 2 and Table 4). When returns are aggregated to the hourly frequency, we find that the leverage effect is statistically significant for the first four hours, while the volatility feedback effect is negligible at all horizons. Using daily observations, derived from high-frequency data, we find a strong leverage effect for the first three days, while the volatility feedback effect appears to be negligible at all horizons. The results based on realized volatility (RV) and bipower variation (BV) are essentially the same. Overall, these results show that the leverage effect is more important than the volatility feedback effect (Figure 2).

If the feedback effect from volatility to returns is almost nonexistent, we find that the instantaneous causality between these variables exists and remains economically and statistically important for several days. This means that volatility has a contemporaneous effect on returns, and similarly returns have a contemporaneous effect on volatility. These results are confirmed with both realized and bipower variations. Furthermore, dependence between volatility and returns is also economically and statistically important for several days.

Let us now consider a trivariate autoregressive model including implied volatility in addition to realized volatility (bipower variation) and returns, as suggested in Section 3 (Figures 3–5). First, we find that implied volatility (IV) helps to predict future realized volatility for *several days ahead* (Figure 3). Many other papers like Day and Lewis (1992), Canina and Figlewski (1993), and Lamoureux and Lastrapes (1993), among others, also find that implied volatility can be used to predict future volatility. However, the difference is that in the present paper we look at h-step-ahead forecast, for $h \geqslant 1$, whereas the previous papers only focus

[12] The HAR-RV scheme, in which the realized volatility is parameterized as a linear function of the lagged realized volatilities over different horizons, has been proposed by Müller et al. (1997) and Corsi (2009). The MIDAS scheme, based on the idea of distributed lags, has been analyzed and estimated by Ghysels, Santa-Clara, and Valkanov (2002).

[13] Detailed results, including confidence bands on the causality measures, are presented in a separate companion document (Dufour, Garcia, and Taamouti 2010, available from one of the authors' Home page [www.jeanmariedufour.com]).

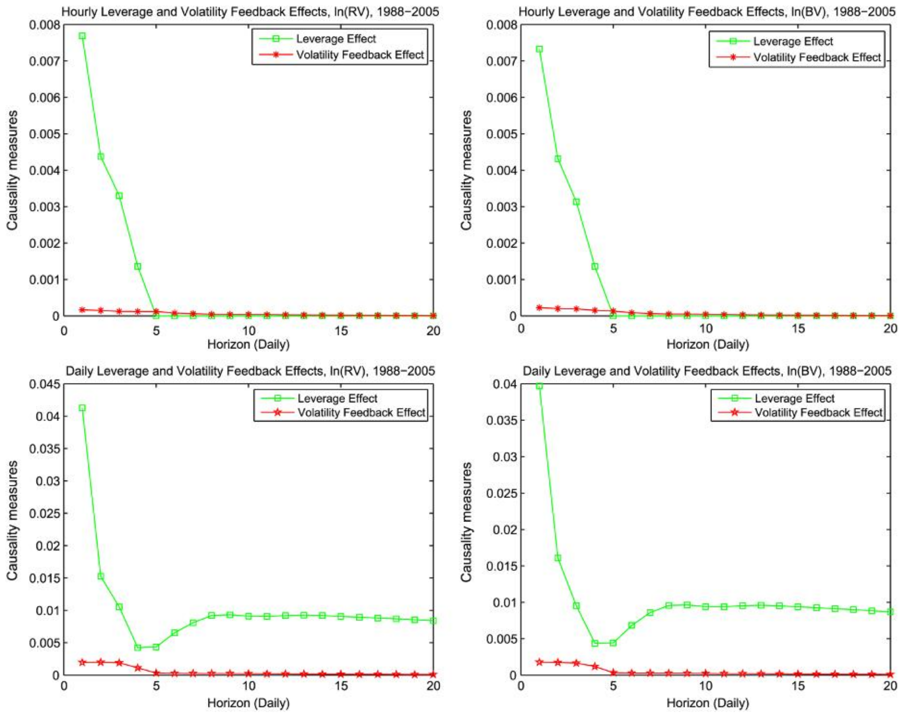

Figure 2 Leverage and volatility feedback effects in hourly and daily data using a bivariate autoregressive model (r, RV). January 1988 to December 2005.

on one-step-ahead forecast. Note that Bollerslev, Litvinova, and Tauchen (2006) do not consider implied volatility in their analysis.

Second, there is an important increase in the volatility feedback effect when implied volatility is taken into account (Figure 4). In particular, it is statistically significant during the first four days. The volatility feedback effect relies first on the volatility clustering phenomena which means that return shocks, positive or negative, increase both current and future volatility. The second basic explanation of this hypothesis is that there is a positive intertemporal relationship between conditional volatility and expected returns. Thus, given the anticipative role of implied volatility and the link between the volatility feedback effect and future volatility, implied volatility reinforces and increases the impact of volatility on returns.[14] Figure 4 also compares volatility feedback effects with and without implied volatility as an auxiliary variable. We see that the difference between IV and RV has a stronger impact on returns than realized volatility alone in the presence

[14]Since option prices reflect market participants' expectations of future movements of the underlying asset, the volatility implied from option prices should be an efficient forecast of future volatility, which potentially explains a better identification of the volatility feedback effect.

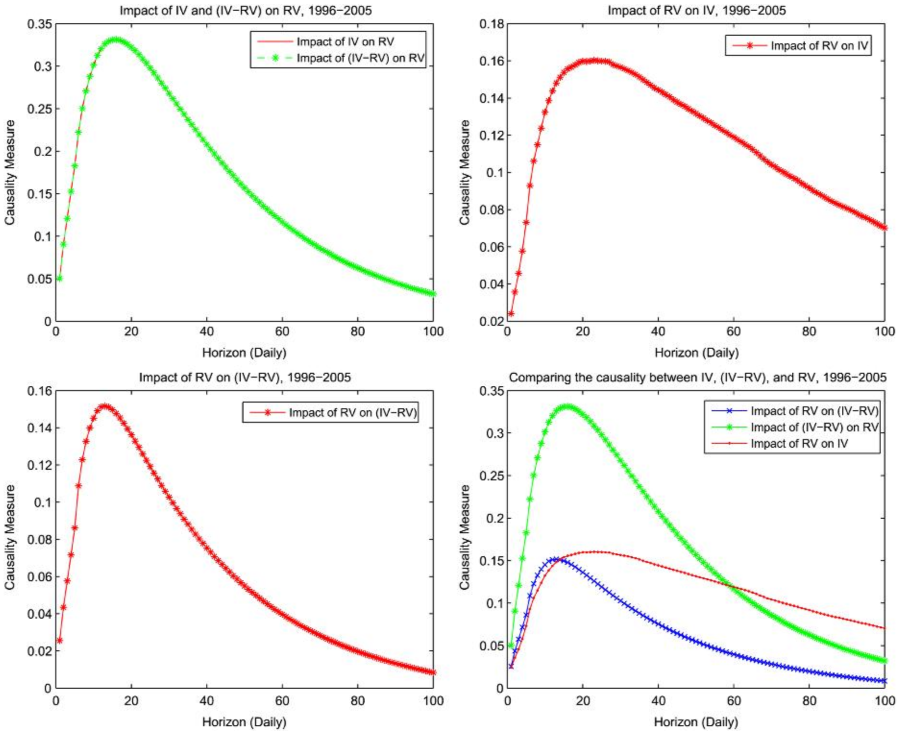

Figure 3 Causality measures between implied volatility (IV) (or variance risk premium IV-RV) and realized volatility (RV), using trivariate VAR models for (r, RV, IV) and (r, RV, IV-RV). January 1996 to December 2005.

of implied volatility. Further, different transformations of volatility (logarithmic of volatility and standard deviation) are considered: the volatility feedback effect is strongest when the standard deviation is used to measure volatility.

Finally, we look at the leverage effects with and without implied volatility as an auxiliary variable (Figure 5). We see that there is almost no change in the leverage effect when we take into account implied volatility. On comparing the leverage and volatility feedback effects with and without implied volatility, we see that the difference, in terms of causality measure, between leverage and volatility feedback effects decreases when implied volatility is included in the information set. In other words, taking into account implied volatility allows to identify a volatility feedback effect without affecting the leverage effect. This may reflect the fact that investors use several markets to carry out their financial strategies, and information is disseminated across several markets. Since the identification of a causal relationship depends crucially on the specification of the information set, including implied volatility appears essential to demonstrate a volatility feedback effect.

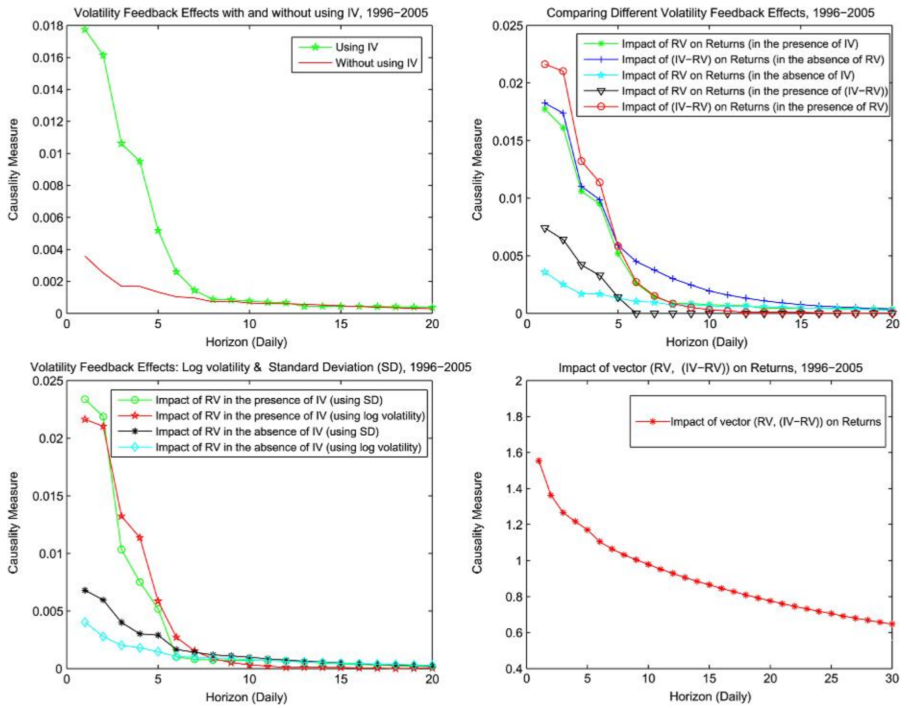

Figure 4 Volatility feedback effects, with implied volatility as auxiliary variable [trivariate models (r, RV, IV) and (r, RV, IV-RV)] and without implied volatility [bivariate model (r, RV)]; different transformations of volatility considered. Impact of vector (RV, IV-RV) on returns. January 1996 to December 2005.

5 DYNAMIC IMPACT OF POSITIVE AND NEGATIVE NEWS ON VOLATILITY

In the previous sections, we did not account for the fact that return news may differently affect volatility depending on whether they are good or bad. We will now propose a method to sort out the differential effects of good and bad news, along with a simulation study showing that our approach can indeed detect asymmetric responses of volatility to return shocks.

5.1 Theory

Several volatility models capture this asymmetry and are explored in Engle and Ng (1993). To study the effect of current return shocks on future expected volatility, Engle and Ng (1993) introduced the NIF. The basic idea of this function is to consider the effect of the return shock at time t on volatility at time $t+1$ in isolation while conditioning on information available at time t and earlier. Recently,

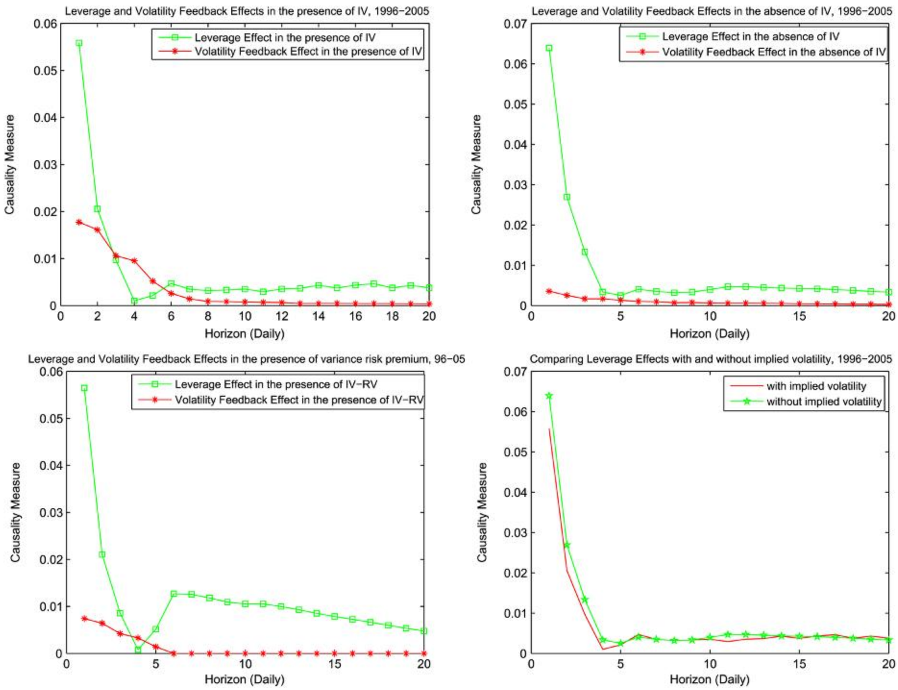

Figure 5 Leverage and volatility feedback effects, with implied volatility as auxiliary variable (trivariate models (r, RV, IV) and (r, RV, IV-RV)) and without implied volatility (bivariate model (r, RV)). January 1996 to December 2005.

Chen and Ghysels (2011) have extended the concept of news impact curves to the high-frequency data setting. Instead of taking a single horizon fixed parametric framework, they adopt a flexible multi-horizon semiparametric modeling (see also Linton and Mammen 2005).

In what follows, we extend our previous VAR model to capture the *dynamic* impact of bad news (negative innovations in returns) and good news (positive innovations in returns) on volatility. We quantify and compare the strength of these effects in order to determine the most important ones. To analyze the impact of news on volatility, we consider the following model:

$$\ln(\sigma_{t+1}^2) = \mu_\sigma + \sum_{j=1}^{p} \varphi_j^\sigma \ln(\sigma_{t+1-j}^2) + \sum_{j=1}^{p} \varphi_j^- er_{t+1-j}^- + \sum_{j=1}^{p} \varphi_j^+ er_{t+1-j}^+ + u_{t+1}^\sigma, \quad (5.1)$$

where

$$er_{t+1-j}^- = \min\{er_{t+1-j}, 0\}, \quad er_{t+1-j}^+ = \max\{er_{t+1-j}, 0\},$$
$$er_{t+1-j} = r_{t+1-j} - \mathsf{E}_{t-j}(r_{t+1-j}),$$

Table 4 Hourly and daily feedback effects

$C(\ln(\text{RV}) \underset{h}{\rightarrow} r)$		$h=1$	$h=2$	$h=3$	$h=4$
Hourly volatility feedback effects using $\ln(\text{RV})$					
Point estimate		0.00016	0.00014	0.00012	0.00012
95% Bootstrap interval		[0.0000, 0.0007]	[0.0000, 0.0006]	[0.0000, 0.0005]	[0.0000, 0.0005]
Hourly volatility feedback effects using $\ln(\text{BV})$					
Point estimate		0.00022	0.00020	0.00019	0.00015
95% Bootstrap interval		[0.0000, 0.0008]	[0.0000, 0.0007]	[0.0000, 0.0007]	[0.0000, 0.0005]
Daily volatility feedback effects using $\ln(\text{RV})$					
Point estimate		0.0019	0.0019	0.0019	0.0011
95% Bootstrap interval		[0.0007, 0.0068]	[0.0005, 0.0065]	[0.0004, 0.0061]	[0.0002, 0.0042]
Daily volatility feedback effects using $\ln(\text{BV})$	Point estimate	0.0017	0.0017	0.0016	0.0011
95% Bootstrap interval		[0.0007, 0.0061]	[0.0005, 0.0056]	[0.0004, 0.0055]	[0.0002, 0.0042]

This table summarizes the estimation results of causality measures from hourly realized volatility [$\ln(\text{RV})$] to hourly returns (r), hourly bipower variation [$\ln(\text{BV})$] to hourly returns, daily realized volatility to daily returns, and daily bipower variation to daily returns, respectively. The point estimate of the causality measures at horizons $h = 1, \ldots, 4$ and the 95% corresponding percentile bootstrap interval are given.

$\mathsf{E}[u_t^\sigma] = 0$, and $\mathsf{Var}[u_t^\sigma] = \Sigma_{u^\sigma}$. Equation (5.1) represents the linear projection of volatility on its own past and the past of centered negative and positive returns. This regression model allows one to capture the effect of centered negative or positive returns on volatility through the coefficients φ_j^- or φ_j^+, respectively, for $j = 1, \ldots, p$. It also allows one to examine the different effects that large and small negative and/or positive information shocks have on volatility. This will provide a check on the results obtained in the literature on GARCH modeling, which has put forward overwhelming evidence on the effect of negative shocks on volatility.

Again, in our empirical applications, σ_{t+1}^2 will be replaced by realized volatility RV_{t+1} or bipower variation BV_{t+1}. Furthermore, the conditional mean return is approximated by the following rolling-sample average:

$$\hat{E}_t(r_{t+1}) = \frac{1}{m}\sum_{j=1}^{m} r_{t+1-j},$$

where we take an average around $m = 15, 30, 90, 120$, and 240 days.[15] Now, let us consider the following restricted models:

$$\ln(\sigma_{t+1}^2) = \theta_\sigma + \sum_{i=1}^{\bar{p}} \bar{\varphi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\bar{p}} \bar{\varphi}_i^+ er_{t+1-j}^+ + e_{t+1}^\sigma, \tag{5.2}$$

$$\ln(\sigma_{t+1}^2) = \bar{\theta}_\sigma + \sum_{i=1}^{\dot{p}} \dot{\varphi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\dot{p}} \dot{\varphi}_i^- er_{t+1-j}^- + v_{t+1}^\sigma. \tag{5.3}$$

Equation (5.2) represents the linear projection of volatility $\ln(\sigma_{t+1}^2)$ on its own past and the past of centered positive returns. Similarly, Equation (5.3) represents the linear projection of volatility $\ln(\sigma_{t+1}^2)$ on its own past and the past of centered negative returns. To compare the forecast error variances of model (5.1) with those of models (5.2) and (5.3), we assume that $p = \bar{p} = \dot{p}$.

Thus, a measure of the impact of bad news on volatility at horizon h, where $h \geqslant 1$, is given by the following equation:

$$C\left(er^- \underset{h}{\rightarrow} \ln(\sigma^2)\right) = \ln\left[\frac{\mathsf{Var}[e_{t+h}^\sigma[\ln(\sigma_{t+h}^2)|(\sigma^2(\omega,t], er^+(\omega,t])]]}{\mathsf{Var}[u_{t+h}^\sigma[\ln(\sigma_{t+h}^2)|J(t)]]}\right],$$

where $e_{t+h}^\sigma[\ln(\sigma_{t+h}^2)|(\sigma^2(\omega,t], er^+(\omega,t])]$ and $u_{t+h}^\sigma[\ln(\sigma_{t+h}^2)|J(t)]$ are the h-step-ahead forecast errors of log volatility based on the information sets $\sigma^2(\omega,t] \cup er^+(\omega,t]$ and $J(t)$ respectively. Similarly, a measure of the impact of good news on volatility at horizon h is given by

$$C\left(er^+ \underset{h}{\rightarrow} \ln(\sigma^2)\right) = \ln\left[\frac{\mathsf{Var}[v_{t+h}^\sigma[\ln(\sigma_{t+h}^2)|(\sigma^2(\omega,t], er^-(\omega,t])]]}{\mathsf{Var}[u_{t+h}^\sigma[\ln(\sigma_{t+h}^2)|J(t)]]}\right],$$

[15] In our empirical application, we also considered the case of uncentered returns. The results can be found in Dufour, Garcia, and Taamouti (2010).

where $v^{\sigma}_{t+h}[\ln(\sigma^2_{t+h})|(\sigma^2(\omega,t], er^-(\omega,t])]$ is the h -step-ahead forecast error of log volatility based on the information set $\sigma^2(\omega,t] \cup r^-(\omega,t]$,

$$er^-(\omega,t] = \{er^-_{t-s}, s \geqslant 0\},$$
$$er^+(\omega,t] = \{er^+_{t-s}, s \geqslant 0\},$$

and $J(t)$ is the information set obtained by "adding" $\sigma^2(\omega,t]$ to $er^-(\omega,t]$ and $er^+(\omega,t]$. We also define a function that allows us to compare the impact of bad and good news on volatility. This function can be defined as follows:

$$C\left(er^-/er^+ \underset{h}{\rightarrow} \ln(\sigma^2)\right) = \ln\left[\frac{\mathsf{Var}[e^{\sigma}_{t+h}[\ln(\sigma^2_{t+h})|(\sigma^2(\omega,t], er^+(\omega,t])]]}{\mathsf{Var}[v^{\sigma}_{t+h}[\ln(\sigma^2_{t+h})|(\sigma^2(\omega,t], er^-(\omega,t])]]}\right].$$

When $C\big(er^-/er^+ \underset{h}{\rightarrow} \ln(\sigma^2)\big) \geqslant 0$, this means that bad news have more impact on volatility than good news. Otherwise, good news have more impact on volatility than bad news. Compared to Chen and Ghysels (2011), our approach is also multi-horizon and based on high-frequency data but is more parametric in nature. Before applying these new measures to our S&P 500 futures market, we conduct a simulation study to verify that the asymmetric reaction of volatility is well captured in various models of the GARCH family that produce or not such an asymmetry.

5.2 Simulation Study on News Asymmetry Detection

We will now present an exploratory simulation study on the ability of causality measures to detect asymmetry in the impact of bad and good news on volatility (Pagan and Schwert 1990; Gouriéroux and Monfort 1992; Engle and Ng 1993). To do this, we consider that returns are governed by a process of the form

$$r_{t+1} = \sqrt{\sigma_t}\varepsilon_{t+1}, \tag{5.4}$$

where $\varepsilon_{t+1} \sim \mathcal{N}(0,1)$ and σ_t represents the conditional volatility of return r_{t+1}. Since we are only interested in studying the asymmetry in leverage effect, Equation (5.4) does not allow for a volatility feedback effect. Second, we assume that σ_t follows one of the following heteroskedastic models:

1. GARCH$(1,1)$ model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\varepsilon^2_{t-1}\ ; \tag{5.5}$$

2. EGARCH$(1,1)$ model:

$$\log(\sigma_t) = \omega + \beta\log(\sigma_{t-1}) + \gamma\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} + \alpha\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}}} - \sqrt{2/\pi}\right]. \tag{5.6}$$

GARCH model is, by construction, symmetric. Thus, we expect that the curves of causality measures for bad and good news will be the same. Similarly, because

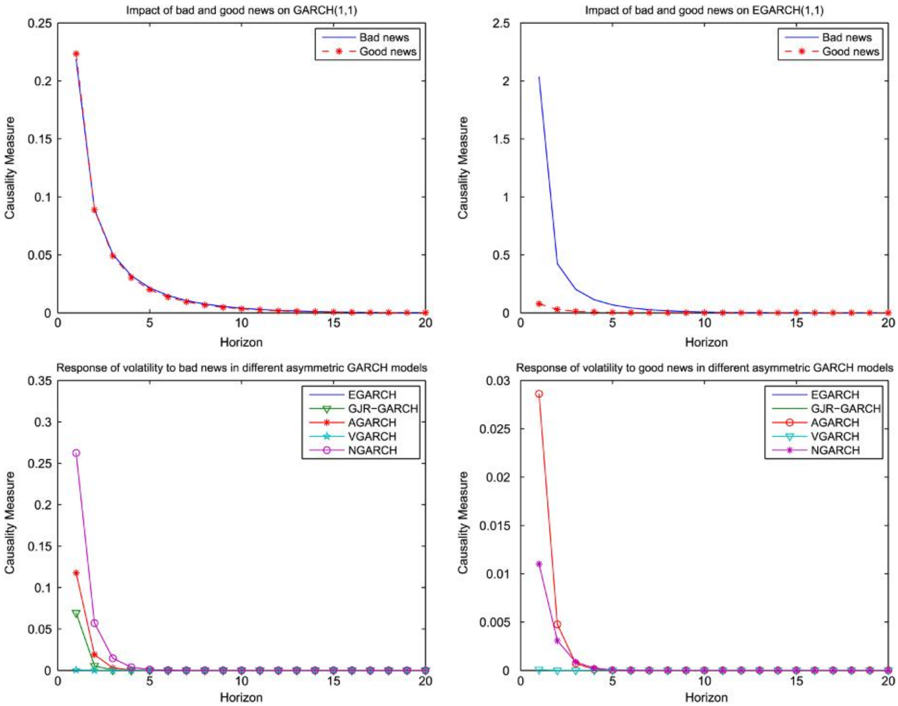

Figure 6 Causality measures of bad and good news on symmetric and asymmetric GARCH volatility models.

EGARCH model is asymmetric, we expect that these curves will be different. The parameter values considered are from Engle and Ng (1993).[16]

To see whether the asymmetric structures get translated into the causality patterns, we then simulate returns and volatilities according to the above models and we evaluate the causality measures for bad and good news as described in Section 5.1. To abstract from statistical uncertainty, the models are simulated with a large sample size ($T = 40,000$).

The results obtained in this way are reported in Figure 6. We see from these that symmetry and asymmetry are well represented by causality measure patterns. For the symmetric GARCH model, bad and good news have the same impact on volatility. In contrast, for the asymmetric EGARCH model, bad and good news exhibit different impact curves. We also considered many other parametric volatility models like AGARCH(1,1), VGARCH(1,1), NL-GARCH, GJR-GARCH, and

[16]These parameters are the results of an estimation of different parametric volatility models using the daily returns series of the Japanese TOPIX index from January 1, 1980, to December 31, 1988. For details, see Engle and Ng (1993). We also considered other values based on Engle and Ng (1993). The results are similar to those presented here (see Dufour, Garcia, and Taamouti 2010).

nonlinear asymmetric GARCH$(1,1)$ [NGARCH(1,1)], and the results correspond to what we were expecting.[17]

It is also interesting to observe for the asymmetric models that bad news have a greater impact on volatility than good news. The magnitude of the volatility response is largest for NGARCH model, followed by the AGARCH and GJR-GARCH models. The effect is negligible in EGARCH and VGARCH models. The impact of good news on volatility is more noticeable in AGARCH and NGARCH models. Overall, causality measures appear to capture quite well the effects of returns on volatility, both qualitatively and quantitatively.

6 NEWS EFFECTS IN S&P 500 FUTURES MARKET

We now apply the good news and bad news measures of causality to S&P 500 futures returns. To carry out our analysis, we consider two alternative measures of news: (i) positive and negative deviations of returns from average past returns, and (ii) positive and negative variance risk premiums. An important feature of our approach comes from the fact that a specific volatility model need not be estimated, which can be contrasted with previous related studies (see, e.g., Bekaert and Wu 2000; Engle and Ng 1993; Glosten, Jagannathan, and Runkle 1993; Campbell and Hentschel 1992; Nelson 1991).

6.1 Return News

Our empirical results on return news effect are summarized and compared in Figure 7. Detailed results (with confidence intervals) are presented in Tables 5–6.[18] We find a much stronger impact of bad news on volatility for several days. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons. So our central finding is that bad news have more impact on volatility than good news at all horizons.

6.2 Variance Risk Premium

Let us now look at the reaction of future returns to the sign of the difference between implied volatility and realized volatility (bipower variation). This difference is a measure of the variance risk premium since the option-implied volatility includes the risk premium that investors associate with expected future volatility (see Bollerslev and Zhou 2005; Drechsler and Yaron 2011). We will therefore assess

[17]See Dufour, Garcia, and Taamouti (2010).

[18]We also computed the causality measures of the impact of bad and good news on volatility using other estimators of the conditional mean ($m = 90, 120, 240$) and uncentered returns. The results are similar to the ones discussed here (see Dufour, Garcia, and Taamouti 2010).

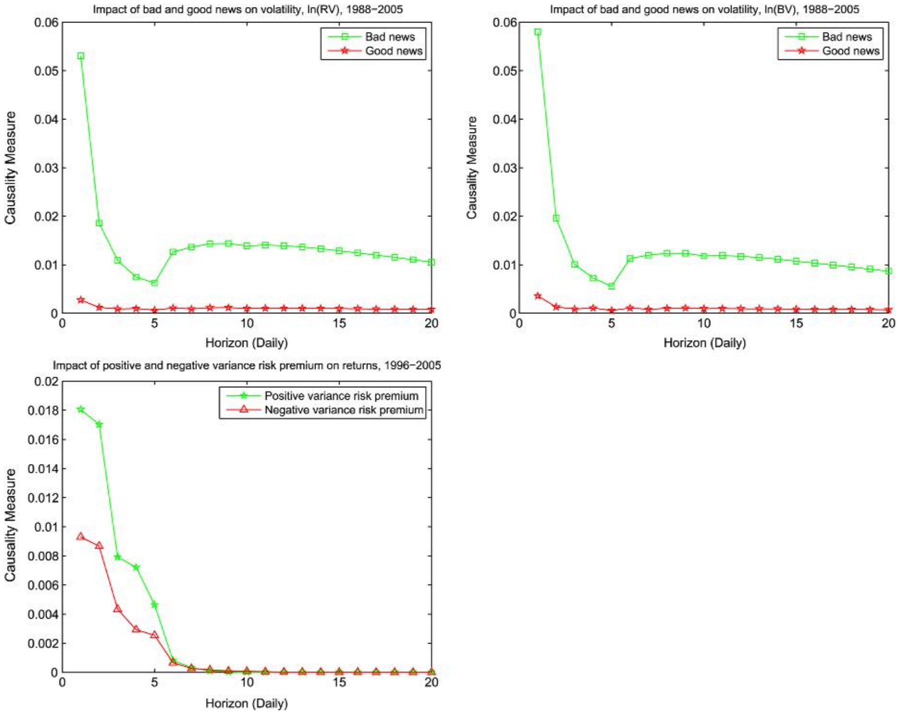

Figure 7 Causality measures of the impact of news (return news and volatility news) on returns and volatility, based on realized volatility (ln(RV)) and bipower variation (ln(BV)). January 1988 to December 2005.

whether a positive variance risk premium has an impact of similar magnitude on expected returns than a negative variance risk premium. In the case of a positive variance risk premium, we expect an increase in the expected returns (return risk premium), and in the opposite, we expect a decrease in expected returns.

Since implied volatility is a predictor of future volatility, we write

$$\ln(\mathrm{RV}_{t+h}) = f(\ln(\mathrm{IV}_t), \ln(\mathrm{IV}_{t-1}), \ldots) + \varepsilon_{t+h}, \quad \forall h \geqslant 1,$$

$$\varepsilon_{t+h} = \ln(\mathrm{RV}_{t+h}) - f(\ln(\mathrm{IV}_t), \ln(\mathrm{IV}_{t-1}), \ldots), \tag{6.1}$$

where $f(\ln(\mathrm{IV}_t), \ln(\mathrm{IV}_{t-1}), \ldots)$ is a function of the past observations on implied volatility.[19] The term on the right-hand side of Equation (6.1) can be viewed as an

[19] $f(\ln(\mathrm{IV}_t), \ln(\mathrm{IV}_{t-1}), \ldots)$ represents the optimal forecast, in the sense of minimization of the mean squared error, of $\ln(\mathrm{RV}_{t+h})$ based on the past observations of implied volatility.

Table 5 Measuring the impact of good news on volatility: centered positive returns, ln (RV)

$C(er^{+} \underset{h}{\rightarrow} \ln(\mathrm{RV}))$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{15}\sum_{j=1}^{15} r_{t+1-j}$				
Point estimate	0.0007	0.0007	0.0007	0.0004
95% Bootstrap interval	[0.0003, 0.0043]	[0.0002, 0.0039]	[0.0001, 0.0034]	[0.0000, 0.0030]
$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{30}\sum_{j=1}^{30} r_{t+1-j}$				
Point estimate	0.0010	0.0007	0.0007	0.0005
95% Bootstrap interval	[0.0004, 0.0051]	[0.0003, 0.0039]	[0.0003, 0.0036]	[0.0000, 0.0032]

This table summarizes the estimation results of causality measures from centered positive returns (er^{+}) to realized volatility [$\ln(\mathrm{RV})$] using two estimators of the conditional mean, for $m = 15, 30$. The point estimate of the causality measures at horizons $h = 1, \ldots, 4$ and the 95% corresponding percentile bootstrap interval are given.

Table 6 Measuring the impact of good news on volatility: centered positive returns, ln(BV)

$C(er^{+} \underset{h}{\rightarrow} \ln(\mathrm{BV}))$	$h=1$	$h=2$	$h=3$	$h=4$
$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{15}\sum_{j=1}^{15} r_{t+1-j}$				
Point estimate	0.0008	0.0008	0.0006	0.0006
95% Bootstrap interval	[0.0003, 0.0045]	[0.0002, 0.0041]	[0.0002, 0.0035]	[0.0000, 0.0034]
$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{30}\sum_{j=1}^{30} r_{t+1-j}$				
Point estimate	0.0012	0.0007	0.0007	0.0007
95% Bootstrap interval	[0.0005, 0.0053]	[0.0003, 0.0041]	[0.0002, 0.0039]	[0.0001, 0.0038]

This table summarizes the estimation results of causality measures from centered positive returns (er^{+}) to bipower variation (ln(BV)) using two estimators of the conditional mean, for $m = 15, 30$. The point estimate of the causality measures at horizons $h = 1, \ldots, 4$ and the 95% corresponding percentile bootstrap interval are given.

approximation of volatility shocks or volatility news. To measure empirically the *dynamic* impact of volatility news on returns, we consider the following model:

$$r_{t+1} = \mu_r + \sum_{j=1}^{p} \varphi_j^r r_{t+1-j} + \sum_{j=1}^{p} \varphi_j^- VP_{t+1-j}^- + \sum_{j=1}^{p} \varphi_j^+ VP_{t+1-j}^+ + u_{t+1}^r, \tag{6.2}$$

where $VP_{t+1-j}^- = \min\{VP_{t+1-j}, 0\}$, $VP_{t+1-j}^+ = \max\{VP_{t+1-j}, 0\}$, and

$$VP_{t+1-j} = \ln(\mathrm{IV}_{t+1-j}) - \ln(\mathrm{RV}_{t+1-j}), \quad j = 1, \ldots, p.$$

Equation (6.2) represents a linear projection of returns on its own past and the past of negative and positive variance risk premiums. This regression allows one to capture the effect of volatility news on returns through the coefficients φ_j^- or φ_j^+, for $j = 1, \ldots, p$. It also allows one to examine different effects that large and small negative and/or positive volatility shocks have on return risk premium. When implied volatility is bigger than realized volatility (bipower variation), we expect an increase in future volatility followed by an increase in the expected returns. In the opposite situation, we expect a decrease in future volatility followed by a decrease in the expected returns.

The empirical results on the impact of volatility news on returns are given in Figure 7.[20] The latter compares the impacts of negative and positive variance risk premium on returns. We see that a positive variance risk premium has more impact on expected returns than a negative variance risk premium, which means that positive shocks on volatility have more impact on returns than negative shocks. The impact is twice as big on the first day and shrinks to zero after about five days. By looking at the sign of coefficients φ_j^+ and φ_j^-, for $j = 1, \ldots, p$, we find that φ_j^+ are positive, whereas φ_j^- are negative, as expected. Consequently, the increase in expected returns tends to be higher than the decrease for a movement in the variance risk premium of the same magnitude but of opposite signs.

7 CONCLUSIONS

In this paper, we analyze and quantify the relationship between volatility and returns with high-frequency equity returns. Within the framework of a VAR linear model of returns and realized volatility or bipower variation, we quantify the dynamic leverage and volatility feedback effects by applying short-run and long-run causality measures proposed by Dufour and Taamouti (2010). These causality measures go beyond simple correlation measures used recently by Bollerslev, Litvinova, and Tauchen (2006).

Using five minute observations on S&P 500 Index futures contracts, we measure a weak dynamic leverage effect for the first four hours in hourly data and a strong dynamic leverage effect for the first three days in daily data. The volatility

[20] Detailed results (with confidence intervals) are presented in Dufour, Garcia, and Taamouti (2010).

feedback effect is found to be negligible at all horizons. Interestingly, when we remeasure the dynamic leverage and volatility feedback effects using implied volatility (IV), we find that a volatility feedback effect appears, while the leverage effect remains almost the same. This can be explained by the power of implied volatility to predict future volatility and by the fact that volatility feedback effect is related to the latter. We also use causality measures to quantify and test statistically the dynamic impact of good and bad news on volatility. First, we assess by simulation the ability of causality measures to detect the differential effect of good and bad news in various parametric volatility models. Then, empirically, we measure a much stronger impact for bad news at several horizons. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons. We introduce a new concept of news based on volatility. This one is defined by the difference between implied volatility and realized volatility (bipower variation). When implied volatility is bigger than realized volatility (bipower variation), it means that the market is expecting an increase in future volatility with respect to current volatility. Our empirical results show that such an expected increase in volatility has a stronger impact on return risk premium than an expected decrease of a similar magnitude.

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