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Consumption and equilibrium asset pricing: An empirical assessment

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Abstract

In the various attempts to solve the equity premium puzzle, the characterization of the utility function has received a lot of attention, along with the postulated nature of the economy. In this paper, we specify and estimate by maximum likelihood over the period 1871–1985 a heteroskedastic joint consumption and dividend Markov endowment process in an exchange asset pricing model. To assess the model, we try to replicate both the first and second unconditional moments of the return series, the negative serial correlation present in real and excess returns and the forecasting power of the dividend-price ratio for multiperiod returns. For the real returns, the model captures to some extent the main features of the data for values of the coefficient of risk aversion below 10. The main failure of the model comes from the excess returns. We also assess the model by inferring the consumption growth that rationalizes the observed stock and safe asset returns, but it is too variable to be plausible.

JEL classification: G12

Keywords: Equilibrium asset pricing; Markov switching model; Equity premium puzzle; Serial correlation in returns; Forecastability of returns

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1. Introduction

In the debate over the efficiency of markets in the financial literature, a number of empirical facts have been put forward as a challenge to the proponents of an equilibrium view of the world. The first and most famous concerns the so-called equity premium puzzle unveiled by Mehra and Prescott (1985): the difference between the return on equity and the return on a risk-free asset is too high historically to be explained in a complete-market Arrow–Debreu equilibrium framework. More recently, other ambivalent empirical facts regarding the dynamics of returns have been detected: there seems to be negative autocorrelation in both real and excess long-horizon returns (Fama and French, 1988a, Poterba and Summers, 1988); the dividend-price ratio seems to have some forecasting power for equity returns (Fama and French, 1988b). The ambivalence of this evidence comes from the fact that, in theory, it can be consistent with both an efficient market and an inefficient market explanation.

The main thrust of the efforts to provide an equilibrium theory consistent with the facts has been aimed at the equity premium puzzle. Researchers tried to reshape the various building blocks of the exchange economy model initially proposed by Mehra and Prescott (1985) [1] in order to come up with pieces that will fit the puzzle. The characterization of the utility function has received a lot of attention, [2] along with the postulated nature of the economy. [3] No such scrutiny has been applied to the representation of the endowment process which characterizes the risk in an exchange economy and has potentially an important role to play in the pricing of assets. In most studies, the parameters of the endowment process are calibrated to fit some moments of the data based on a finite state Markov-chain approximation.

In this paper, we therefore pay careful attention to the modeling and estimation of the endowment process. Given the skewness and kurtosis present in the actual consumption and dividend growth series, [4] we assume a heteroskedastic joint bivariate Markov endowment process and estimate by maximum likelihood various models in this class, based on the methodology of Hamilton (1989). Over the

[1] Existence of equilibrium in this type of exchange economy, where the growth rate of consumption follows a Markov process, has been proven in Mehra (1988). In the Lucas (1978) exchange economy, it is the level of consumption that follows a Markov process.

[2] Most notably, Epstein and Zin (1991) and Weil (1989) explore the implications of abandoning the equality between the coefficient of relative risk aversion and the inverse of the intertemporal elasticity of substitution imposed by the usual choice of a time-separable Von Neumann–Morgenstern utility and a power function; Constantinides (1990) abandons the time separability of utility and builds a model of habit formation.

[3] Tauchen (1986) and Kocherlakota (1988) abandon the original Lucas assumption of a singular joint distribution for consumption and dividends; Kandel and Stambaugh (1990) propose a leveraged economy with a risky bond where equity becomes a residual claim on output; Rouwenhorst (1995), Abel (1994), and Cochrane (1991) construct a production economy.

[4] This is documented in Cecchetti et al. (1990).

period 1871–1985, data provide support for a three-state model. In the assessment of the model, we take a wider scope than the equity premium and try to replicate both the first and second unconditional moments of the return series, the negative serial correlation present in real and excess returns and the forecasting power of the dividend-price ratio for multiperiod returns.

The idea of disentangling the consumption and dividend processes originates with Tauchen (1986) and has been used by various authors to build asset pricing models. For example, Abel (1994) provides an equilibrium model where output (which equals consumption) is the sum of labour income and capital income (dividends). Danthine and Donaldson (1995) also build a general equilibrium model where consumption is no longer constrained to equal dividends. Empirically, the distinction between consumption and dividends should matter given the very different characteristics of the series. Dividends are much more variable than consumption and both series seem to exhibit heteroskedasticity. As equilibrium return formulas in Section 2 will make clear, heteroskedasticity affects both the level and the variance of equilibrium returns. Since dividends are more variable than consumption and affect only equity returns, the equilibrium equity premium could be higher than with a homoskedastic process. Moreover, while the ARCH literature has paid considerable attention to the modeling of conditional second moments of asset returns, asset pricing models have surprisingly neglected the second moments to put the emphasis on the first moment. Most of the literature assessing the empirical validity of asset pricing models revolves around the equity premium puzzle. Ideally, however, a good model should explain the whole distribution function of returns. Our paper enlarges the set of criteria used to assess the empirical validity of asset pricing models in the hope of better understanding where the successes and failures of the model come from.

Table 1 illustrates, for the period 1871–1985, the empirical facts related to asset returns that we will try to reproduce with our model. [5] Part A presents the first and second moments of the one-year Treasury Bill, the equity return and the equity premium series. Over this period, the risk premium averaged 6.1% with a standard deviation of 18.6% and a correlation with the Treasury Bill rate of −0.1165. The Treasury Bill rate averaged 2.02% with a standard deviation of 6.43%. In Part B, there seems to be evidence of negative serial correlation in real returns at various lags. This negative autocorrelation seems also to be present in excess returns, although the regression coefficients start to be positive at lags 5 and higher. These point estimates suggest strong predictability. [6] Finally, in Part

[5] See data sources in Appendix A.

[6] However, Poterba and Summers (1988) cannot reject, based on returns variance ratio tests, the hypothesis that stock prices follow a random walk and Fama and French (1988b) and Kim et al. (1991) show that the mean reversion in prices implied by such negative autocorrelation in returns is mainly a feature of the 1926–1946 period. The problem of assessing the significance of the variance ratios and autocorrelation of multiyear returns is discussed further in Richardson (1990) and Richardson and Stock (1989).

Table 1

A: First and second moments of returns

	Mean (%)	Standard deviation (%)
Treasury Bill rate (R_f)	2.02	6.43
Equity return (R_q)	8.13	18.93
Equity premium (R_p)	6.11	18.57
Correlation (R_f, R_q)		0.2252
Covariance (R_f, R_p)		−0.1165

B: Measures of serial correlation

	Variance ratios $VR = Var(R_{t,t+k})/k Var(R_t)$		Regression coefficients $R_{t,t+k} = a + bR_{t-k,t} + \epsilon_t$	
k	Real returns	Excess returns	Real returns	Excess returns
1	1.0000	1.0000	0.0188	0.0871
2	1.0256	1.0957	−0.1539	−0.1142
3	0.9044	1.0012	−0.1432	−0.1668
4	0.8785	0.9849	−0.1376	−0.2128
5	0.8572	0.9361	−0.1601	−0.1680
6	0.7941	0.8509	−0.0957	0.0082
7	0.7625	0.7906	−0.1037	0.1133
8	0.7781	0.7795	−0.1982	0.1025
9	0.7646	0.7710	−0.2792	0.0841
10	0.7395	0.7828	−0.3829	0.0606

C: Return forecastability based on dividend yield regression: $R_{t,t+k} = \alpha + \beta(D_t/P_t) + u_{t,t+k}$

	Real returns				Excess returns			
k	Coef.	t	R^2	σ	Coef.	t	R^2	σ
1	2.87	1.91	0.031	0.19	3.46	2.35	0.047	0.18
2	5.31	2.48	0.053	0.27	6.45	3.01	0.076	0.27
3	7.05	2.89	0.071	0.30	8.29	3.36	0.092	0.31
4	10.10	3.72	0.113	0.34	11.16	3.99	0.127	0.35
5	13.59	4.68	0.169	0.36	14.51	4.92	0.183	0.36

C, the regressions of multiperiod returns (one to five years) on current dividend yield (dividend-price ratio) produce positive and significant coefficients, increasing with the horizon. [7] This is true for both real and excess returns.

[7] Fama and French (1988b) adjust the standard errors in the t-statistics for the sample autocorrelation of overlapping residuals. We do not make this adjustment because we want simply reference values for our simulated t-statistics. For the statistical properties of the OLS estimator in the regression of multiperiod returns on dividend yields, see Hodrick (1991).

As a general assessment of the results, we can say that, for real returns, the model captures to some extent the main features of the data for values of the coefficient of risk aversion below 10. In particular, the heteroskedastic nature of the joint consumption-dividends process allows the model to reproduce closely the second moments of the stock and risk-free asset returns. While Cecchetti et al. (1993), who used a two-state homoskedastic joint process for consumption and dividends, obtained similar results for the first and second moments of the stock returns, they could not reproduce the second moment of the risk-free asset. Moreover, their model cannot replicate neither the negative serial correlation of real returns [8] nor the forecastability of multiperiod returns by the dividend yield. No equilibrium asset pricing model has until now reproduced the latter empirical fact.

The main failure of the model comes from excess returns: the equity premium is, as in Mehra and Prescott (1985), hopelessly too low; no evidence of negative autocorrelation is found in excess returns; the dividend-price ratio has no forecasting power at all for excess returns.

One justification for this general failure is to argue that aggregate consumption is too smooth to rationalize the observed risk premia. We therefore ask in the last section the following question: What would be the parameters of the assumed Markov endowment process that will rationalize the returns observed on the market for the risk-free rate and the equity returns?

This approach is in the spirit of Hansen and Jagannathan (1991) except that our inference is based on the particular parametric specification chosen in our model. We use the return equations implied by the model and the assumed specification for the dividend process to estimate by maximum likelihood the equity price-dividend ratios and risk-free asset prices from the actual returns and dividends. Assuming values for the remaining parameters (coefficient of risk aversion, discount factor and correlation between consumption and dividends innovations), we can infer the consumption parameters that will rationalize the observed returns.

In Section 2, a synthetic version of the asset pricing model is developed. In Section 3, the parameters of the model are estimated by maximum likelihood. The model ability to replicate the stylized facts is assessed in Section 4. Section 5 infers the consumption parameters that rationalize the observed returns. Section 6 concludes and outlines future possible avenues of research.

[8] Another justification for the choice of a bivariate process is that a model based on either one of these series as characterizing the endowment process is unable to produce the kind of negative autocorrelation detected in the return data. Bonomo and Garcia (1994) show that the univariate model chosen for the endowment process (a two-state Markov model with a very low mean in one of the states) by Cecchetti et al. (1990) in an identical exchange model is not the best in the class of Markov models and that once the proper specification (with one mean and two variances) is selected, the mean reversion effect disappears.

2. The asset pricing model

Many identical infinitely-lived agents maximize their intertemporal utility and receive each period an endowment of a nonstorable good. Assuming additive time separability of the utility function and a constant discount factor β, the representative agent utility at time t, V_t, can be written as:

$$V_t = E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}), \tag{1}$$

where E_t denotes expectation conditional on information available at time t and C_t the per capita consumption. We assume that $U(\cdot)$ is the power utility function with constant relative risk aversion γ:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}. \tag{2}$$

Equilibrium requires that the prices of existing assets are such that the representative agent is satisfied to consume her expected endowment. In other words, the agent cannot increase her utility by changing the expected intertemporal allocation of her endowment through the purchase or sale of an asset or portfolio of assets.

Defining equity in this economy as an asset that gives as payoff a dividend D_t, its price (P^e) in equilibrium should satisfy the following first-order condition:

$$P_t^e U'(C_t) = \beta E_t U'(C_{t+1}) \left[P_{t+1}^e + D_{t+1} \right]. \tag{3}$$

Using the power utility function defined in Eq. (2), this condition can be rewritten as:

$$P_t^e = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left[P_{t+1}^e + D_{t+1} \right]. \tag{4}$$

In Tauchen (1986), the consumption endowment is not equal to dividends and includes payoffs from other assets unspecified in the model. This separation of consumption and dividends does not take away any generality from the model and has the advantage to make it more realistic given the observed different behavior for consumption and dividend growth.

The price-dividend ratio for the equity is then given by:

$$\frac{P_t^e}{D_t} = \sum_{j=1}^{\infty} \beta^j E_t \left(\frac{C_{t+j}}{C_t} \right)^{-\gamma} \frac{D_{t+j}}{D_t}. \tag{5}$$

Similarly, in this economy, one can define a riskless asset which pays one unit of consumption with certainty the next period. In equilibrium, its price (P^f) will satisfy the following first-order condition:

$$P_t^f U'(C_t) = \beta E_t U'(C_{t+1}). \tag{6}$$

Using Eq. (2), the price of the risk free asset can therefore be written as:

$$P_t^f = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{7}$$

We postulate that the logarithms of consumption and dividends follow a bivariate random walk where both the means and the variances change stochastically according to a Markov variable S_t which takes the values 0, 1,..., $K-1$ (in the case of K states). The sequence $\{S_t\}$ of Markov variables evolves according to the following transition probability matrix Π:

$$\Pi = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0(k-1)} \\ p_{10} & p_{11} & \cdots & p_{1(k-1)} \\ \vdots & \vdots & \vdots & \vdots \\ p_{(k-1)0} & p_{(k-1)1} & \cdots & p_{(k-1)(k-1)} \end{bmatrix}. \tag{8}$$

The bivariate consumption-dividends process can then be written as:

$$\begin{aligned} c_t - c_{t-1} &= \alpha_0^c + \alpha_1^c S_{1,t} + \ldots + \alpha_{k-1}^c S_{k-1,t} \\ &\quad + \left(\omega_0^c + \omega_1^c S_{1,t} + \ldots + \omega_{k-1}^c S_{k-1,t} \right) \epsilon_t^c, \\ d_t - d_{t-1} &= \alpha_0^d + \alpha_1^d S_{1,t} + \ldots + \alpha_{k-1}^d S_{k-1,t} \\ &\quad + \left(\omega_0^d + \omega_1^d S_{1,t} + \ldots + \omega_{k-1}^d S_{k-1,t} \right) \epsilon_t^d, \end{aligned} \tag{9}$$

where $S_{i,t}$ is a function of the state of the economy, S_t, taking value 1, whenever $S_t = i$ and 0 otherwise; c_t and d_t are respectively $\ln C_t$ and $\ln D_t$; ϵ_t^c and ϵ_t^d are $N(0,1)$ error terms with correlation ρ_{cd}. Then, in state i, the means and standard deviations of the growth rates of consumption and dividends will be given respectively by $(\alpha_0^c + \alpha_i^c, \omega_0^c + \omega_i^c)$ and $(\alpha_0^d + \alpha_i^d, \omega_0^d + \omega_i^d)$.

Given the joint process defined by Eq. (9) and the transition probability matrix Π, we can find closed form solutions for the asset prices and easily derive the formulas for returns.

Iterating each equation n times in the system (Eq. (9)), we obtain:

$$\begin{aligned} \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma} &= \exp\left(-\gamma \sum_{j=1}^{n} \alpha_0^c + \alpha_1^c S_{1,t+j} + \ldots + \alpha_{k-1}^c S_{k-1,t+j} \right. \\ &\quad \left. + \left(\omega_0^c + \omega_1^c S_{1,t+j} + \ldots + \omega_{k-1}^c S_{k-1,t+j} \right) \epsilon_{t+j}^c \right). \end{aligned} \tag{10}$$

$$\begin{aligned} \left(\frac{D_{t+n}}{D_t} \right) &= \exp\left(\sum_{j=1}^{n} \alpha_0^d + \alpha_1^d S_{1,t+j} + \ldots + \alpha_{k-1}^d S_{k-1,t+j} \right. \\ &\quad \left. + \left(\omega_0^d + \omega_1^d S_{1,t+j} + \ldots + \omega_{k-1}^d S_{k-1,t+j} \right) \epsilon_{t+j}^d \right). \end{aligned} \tag{11}$$

Multiplying Eq. (10) by Eq. (11), taking the conditional expectation with respect to the information set at time t and using the independence of the sequences $\{S_t\}$ and $\{\epsilon_t^c, \epsilon_t^d\}$, we obtain: [9]

$$E_t\left(\frac{C_{t+n}^{-\gamma} D_{t+n}}{C_t^{-\gamma} D_t}\right) = E_t \exp(\mu_0 n + \mu_1 i_{1t,n} + \ldots + \mu_{k-1} i_{k-1t,n}), \tag{12}$$

where:

$$\mu_0 = -\gamma\alpha_0^c + \alpha_0^d + \frac{1}{2}\left(\gamma^2 \omega_0^{c\,2} + \omega_0^{d\,2} - 2\gamma\omega_0^c \omega_0^d \rho_{cd}\right),$$

$$\mu_j = -\gamma\alpha_j^c + \alpha_j^d + \frac{1}{2}\Big[\gamma^2 \omega_1^{c\,2} + \omega_j^{d\,2} - 2\gamma\left(\omega_j^d \omega_0^c + \omega_0^d \omega_j^c + \omega_j^d \omega_j^c\right)\rho_{cd},$$

$$+2\gamma^2 \omega_0^c \omega_j^c + 2\omega_0^d \omega_j^d\Big] \quad j = 1, \ldots, k-1,$$

$$i_{jt,n} = \sum_{h=1}^{n} S_{j,t+h} \quad j = 1, \ldots, k-1. \tag{13}$$

The expectation term on the right hand side of Eq. (12) can be written in matrix form:

$$E_t \exp(\mu_0 n + \mu_1 i_{1t,n} + \ldots + \mu_{k-1} i_{k-1t,n}) = I_{k,t} A^n \boldsymbol{l}, \tag{14}$$

where $I_{k,t}$ is a $k \times 1$ row vector with 1 in the column corresponding to the state at time t and zeros in the other columns, $\boldsymbol{l}$ is a $k \times 1$ column vector of ones and the matrix A is given by:

$$A = \Pi M \text{ with: } M = \operatorname{diag}(e^{\mu_0}, e^{\mu_0 + \mu_1}, \ldots, e^{\mu_0 + \mu_{k-1}}). \tag{15}$$

So, expression (11) can be substituted in the equity price-dividend Eq. (5) to obtain the following formula:

$$P_t^e = D_t \rho(S_t), \tag{16}$$

where $\rho(S_t)$ is given by:

$$\rho(S_t) = I_{k,t}\left[(I - \beta A)^{-1} - I\right] \boldsymbol{l}. \tag{17}$$

Similarly, the price for the risk-free asset will be given by:

$$\phi(S_t) = \beta I_{k,t} \Pi W \boldsymbol{l}, \tag{18}$$

where:

$$W = \operatorname{diag}(e^{w_0}, e^{w_0 + w_1}, \ldots, e^{w_0 + w_{k-1}}) \text{ with:}$$

$$w_0 = -\gamma\alpha_0^c + \frac{1}{2}\left(\gamma^2 \omega_0^{c\,2}\right),$$

$$w_j = -\gamma\alpha_j^c + \frac{\gamma^2}{2}\left(2\omega_0^c \omega_j^c + \omega_j^{c\,2}\right) \quad j = 1, \ldots, k-1. \tag{19}$$

[9] We assume that S_t belongs to the information set at time t.

The return formulas can now be easily derived. For the safe asset, the return will be:

$$R_t^f = \frac{1}{\phi(S_t)}. \tag{20}$$

For the equity, the one-period gross return will be:

$$R_t^e = \frac{P_{t+1}^e + D_{t+1}}{P_t^e} = \left(\frac{P_{t+1}^e + D_{t+1}}{D_{t+1}}\right)\left(\frac{D_t}{P_t^e}\right)\left(\frac{D_{t+1}}{D_t}\right),$$

$$R_t^e = \frac{\rho(S_{t+1}) + 1}{\rho(S_t)} \exp\big(\alpha_0^d + \ldots + \alpha_{k-1}^d S_{k-1,t+1} + \left(\omega_0^d + \ldots + \omega_{k-1}^d S_{k-1,t+1}\right)\epsilon_{t+1}^d\big). \tag{21}$$

These formulas will allow us in Section 4 to compute the theoretical moments implied by the model, but also to generate the distributions of the measures of serial correlation (variance ratios and regression coefficients) and of the coefficients, R-square and t-statistics in the returns on dividend yield regressions. First, however, we need values for the mean (α), standard deviation (ω), and probability (p) parameters. In Section 3, we estimate these values by maximum likelihood.

3. Maximum likelihood estimation of the model parameters

When Mehra and Prescott (1985) tried to reproduce the equity premium in an equilibrium framework, they assumed that consumption was equal to dividends. Given that assumption, they chose to limit the number of Markov states to two, to constrain the transition probabilities to be equal in both states and to set the ω parameters to zero. They selected the remaining parameters so that the average growth rate of per capita consumption and the standard deviation and first-order serial correlation of the growth rate matched the sample values for U.S. consumption growth over the period 1889–1978. This calibration procedure has the advantage to be simple but imposes serious restrictions on the parameter values. To be sure that the model rejection does not come from a misspecification of the endowment process, one has to be careful about choosing a model that fits the data as closely as possible. To be tractable though, the model should allow to find closed-form solutions for the asset returns.

In our estimation, we do not impose any restrictions on the parameters and we let the data determine the number of states, except that in practice the number of available observations limits us to a maximum of three states (which requires already the estimation of 19 parameters). We therefore estimated the bivariate model with two states and three states over the period 1889–1985 (see data sources in Appendix A). The real per capita consumption and dividend growth

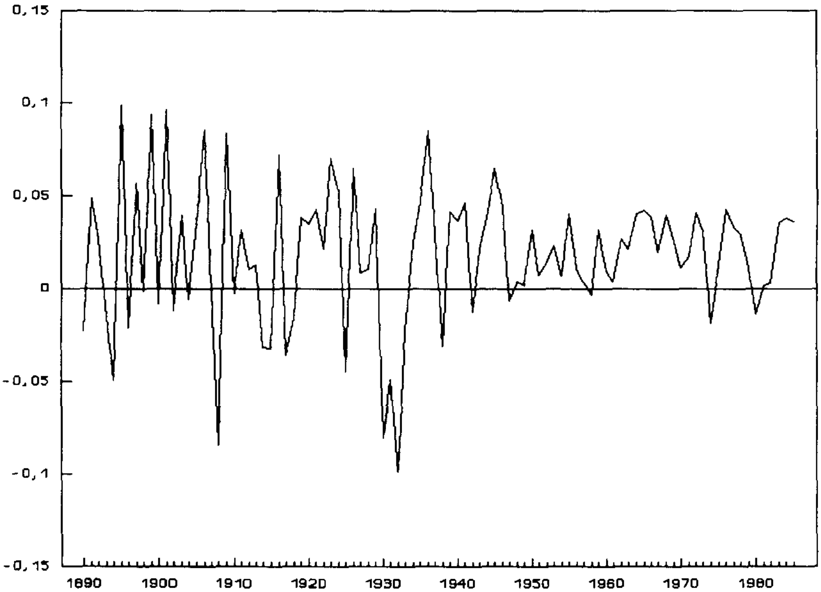

Fig. 1. Real consumption growth.

rates (Figs. 1 and 2 respectively) are computed by differencing the logarithm of the real consumption and dividend series [10] divided by the population. Results are reported in Table 2.

Apart from the mean parameter in the dividend equation, the parameters introduced for the new state seem to be significant, but the standard t-values cannot be used to assess the significance of the parameters. The problem comes from two sources: under the null hypothesis, some parameters are not identified and the scores are identically zero. [11] Recently, Hansen (1992) used empirical process theory to derive a bound for the asymptotic distribution of a standardized likelihood statistic under these two non-standard conditions. Under certain assumptions, Garcia (1995) derives analytically the asymptotic null distribution of the

[10] The real dividend series is obtained by dividing the nominal dividend series by the consumer price index.

[11] To clarify these two irregularities, let us take the case where one wants to test the null hypothesis of a linear model against the alternative hypothesis of a two-state homoskedastic Markov switching model. The null hypothesis can be expressed as either $\{\alpha_1 = 0\}$ or $\{p = 0\}$ or $\{p = 1\}$. To see the problem of unidentified parameters under the null, note that if $\{\alpha_1 = 0\}$, the transition probability parameter p is unidentified since any value between 0 and 1 will leave the likelihood function unchanged. As for the problem of identically zero scores, note that under $\{p = 1\}$, the scores with respect to p, q and α_1 will be identically zero under the null and the asymptotic information matrix will be singular (see Garcia (1992) for more details).

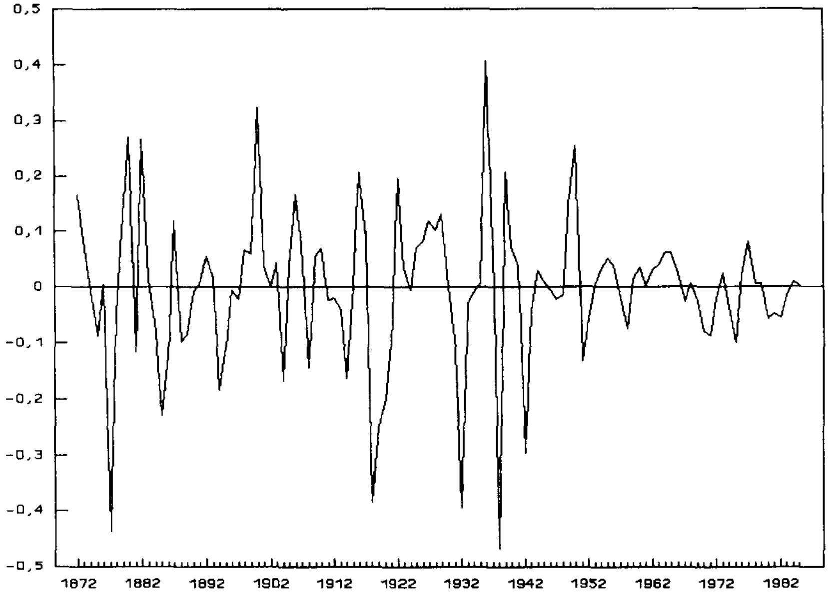

Fig. 2. Real dividend growth.

likelihood ratio test for two-state MS models, based on Hansen's distributional theory for models where nuisance parameters are not identified under the null (Hansen, 1991). Garcia (1995) obtains critical values for the likelihood ratio statistic for a null of a one-state model against two states, which cannot be used here since our null is a bivariate two-state model and our alternative a three-state model. Although the methodology used in Garcia (1995) could be extended to the two-state against three-state case, it has not been done yet. In Garcia and Perron (1996), several other tests are proposed. We report in Table 2 the Davies (1987) test [12] result, which is an upper bound for the p-value of a null of two states. Its value close to zero strongly supports the three-state model. [13]

Looking at the estimation results for the means and variances, we can label state 1 as the bad state, since consumption growth has a low mean (1.3%) and a high standard deviation (4.8%) and dividend growth a high standard deviation (18.3%). Note that the mean of the dividend growth rate is not significantly different from zero in all three states. In the second state, dividend growth is still

[12] See Appendix A in Garcia and Perron (1996) for a description of the test.

[13] Bonomo and Garcia (1994) compute the empirical distribution of the likelihood ratio statistic in a univariate context for a null of a two-state and an alternative of a three-state, where both the means and the variances vary with the state. According to this empirical distribution, the value of 26.60 obtained for the likelihood ratio will translate into a p-value of 0.09.

Table 2
Estimation results for the consumption-dividend joint Markov models (1889–1985)

	Two-state model		Three-state model	
	Coefficient estimate	Standard error	Coefficient estimate	Standard error
α_0^c	0.0238	0.0039	0.0355	0.0075
α_1^c	−0.0091	0.0085	−0.0228	0.0112
α_2^c	-	-	−0.0162	0.0083
ω_0^c	0.0237	0.0029	0.0330	0.0054
ω_1^c	0.0242	0.0057	0.0154	0.0075
ω_2^c	-	-	−0.0167	0.0057
α_0^d	−0.0028	0.0067	−0.0040	0.0069
α_1^d	0.0019	0.0267	0.0053	0.0278
α_2^d	-	-	−0.0011	0.0115
ω_0^d	0.0415	0.0047	0.0204	0.0059
ω_1^d	0.1337	0.0195	0.1629	0.0196
ω_2^d	-	-	0.0267	0.0085
p_{01}	-	-	0.3395	0.1290
p_{02}	-	-	0.00	0.00
p_{11}	0.905	0.072	0.8539	0.0700
p_{12}	-	-	0.0202	0.0201
p_{21}	-	-	0.00	0.00
p_{22}	0.895	0.057	0.9568	0.0359
ρ_{cd}	0.4119	0.087	0.506	0.077
L	455.17		468.47	

Davies test for two states vs. three states: 0.00169 (upper bound of p-value for a null hypothesis of two states).

fairly variable (standard deviation of 4.7%), but consumption growth has an intermediate mean (1.9%) and a low standard deviation (1.6%). Finally, in state 0, dividend growth has the smallest standard deviation (2%), while consumption growth has a high mean (3.6%) and an intermediate standard deviation (3.3%).

The probability parameters indicate that state 2 is the most persistent (p_{22} = 0.957), followed by state 1 (p_{11} = 0.854) and by state 0 (p_{00} = 0.661). The unconditional probabilities π_0, π_1 and π_2 [14] indicate how often in the limit the respective states are reached. State 2 is certainly the most visited state (0.815), while state 1 (the bad state) has a 0.129 probability to be reached. State 0 is the least likely to be visited (0.056).

The estimation algorithm gives as a by-product the probabilities of being in the various states at each point in the sample, given the information available at that

[14] The unconditional probabilities are defined by: $\pi_i = C_{ii} / \Sigma_{j=0}^{k-1} C_{jj}$, where C_{ii} denotes the ith cofactor of the matrix $C = I - \Pi$, with I a $K \times K$ identity matrix and Π as defined in Eq. (8) in the text.

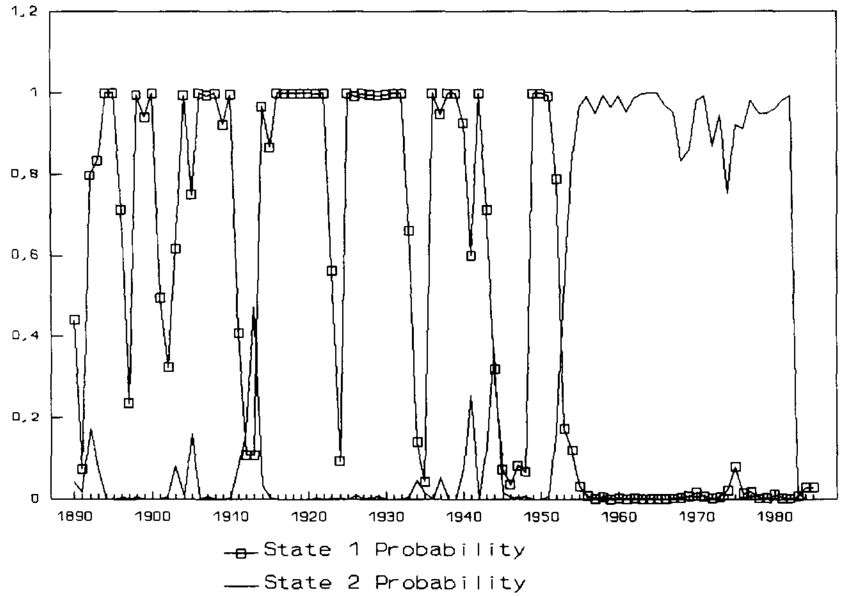

Fig. 3. Filter probabilities ($P(S_t = i \mid Y_t)$: Joint consumption – dividend process.

point. These so-called filter probabilities [15] are shown in Fig. 3. State 2 is mainly reached between the late fifties and the end of the sample, while state 1 is mostly present before the fifties. State 0 occurs at numerous occasions, for lapses of a year or two.

4. Assessing the model according to various measures

Given the theoretical formulas derived in Section 2 for the safe asset and equity returns and the values estimated in Section 3 for the endowment parameters, we are now able to derive and compute the unconditional moments of returns implied by the model and to generate series of returns on which variance ratios, regression coefficients and other statistics can be calculated. Our objective is to give the model a thorough assessment. Most of the efforts in the literature have been geared towards finding a general equilibrium asset pricing model that will solve the equity premium puzzle. But the equity premium is only one statistic and as we have seen in Section 1, there are other stylized facts that can serve as benchmarks for an assessment of the model. As mentioned before, Cecchetti et al. (1990) did a

[15] To know how these filter probabilities are calculated, see Hamilton (1989).

similar exercise with measures of serial correlation in returns, but it can be argued that univariate tests have low power and, therefore, using them as a benchmark might not be very useful. A bivariate test involving returns and dividend yields should be more satisfying to assess the predictability of returns at various horizons. We therefore run regressions of multiperiod returns on dividend yields and add the corresponding statistics (regression coefficients, Student-t and R^2) to the previous criteria to get a full array of measures upon which we can base our assessment of the model.

The analysis proves very useful since all three series of measures point to the same deficiency of the model: its inherent inability to replicate the empirical facts involving excess returns. The equity premium mean is too low, there is no negative autocorrelation in excess returns, and the dividend-price ratio has no forecasting power for excess returns. However, the model performs reasonably well in reproducing the mean and variance of the equity return, the variance of the risk-free rate, the negative autocorrelation in real returns, and the forecastability of real returns by the dividend yield. The following three sub-sections provide a detailed analysis of the results.

4.1. The unconditional moments

Table 3 presents the results for the first and second unconditional moments of the equity return, the risk-free rate and the equity premium for three values of γ, the coefficient of relative risk aversion (or the inverse of the elasticity of intertemporal substitution). We choose a range of 1.5 to 10 in order to stay within the bounds originally set by Mehra and Prescott (1985). We keep a fixed value of

Table 3
Population moments for the risk free return and the equity premium implied by the model

	Actual	$\gamma = 1.5$ $\beta = 0.97$	$\gamma = 5$ $\beta = 0.97$	$\gamma = 10$ $\beta = 0.97$
Mean equity return	8.13%	6.29%	13.48%	23.04%
Standard deviation equity return	18.93%	8.89%	9.80%	13.66%
Mean risk free asset	2.02%	6.14%	12.98%	21.96%
Standard deviation risk free asset	6.43%	0.82%	1.83%	5.44%
Mean risk premium	6.11%	0.15%	0.50%	1.08%
Standard deviation risk premium	18.57%	8.8%	9.7%	12.98%
Correlation risk-free equity return	0.2252	0.0333	0.1349	0.3225
Correlation risk-free risk premium	−0.1165	−0.0129	−0.0526	−0.0793

0.97 for the discount factor β since this parameter does not affect the results in a significant way if it is below unity. [16]

The results are not surprising. For low values of γ, the model is far away from the actual values on all scores. For a γ of 10, the mean of the equity premium is still desperately low (1.08%), but the results are better in terms of second moments. The standard deviation of the equity premium is close to 14%, compared to an actual of 18.5%, and the standard deviation of the risk-free rate is about 1% lower than the actual of 6.4%. For the correlation coefficient between the risk-free rate and the equity return we obtain 0.32 compared with an actual value of 0.225, while for the correlation between the risk-free rate and the equity premium the value produced by the model is -0.079 compared with -0.1165 for the actual.

Cecchetti et al. (1993) also use a joint Markov switching model for consumption and dividends, but they constrain the number of states to two and the variances to be the same in both states. While this specification produces results comparable to ours for all the means and for the standard deviation of the equity return, it fails completely to reproduce the standard deviation of the risk-free rate. Moreover, it also fails to reproduce the negative serial autocorrelation and the forecastability of multiperiod returns by the dividend yield for the real returns, [17] contrary to our model as we will see below.

With respect to the unconditional moments of the return series, we can therefore conclude that allowing for the dividends to be different from aggregate consumption in an exchange economy does not go any length in resolving the equity premium puzzle, but that our heteroskedastic specification for the joint Markov switching model of consumption and dividends seems however to reproduce quite well the unconditional second moments of both the equity return and the risk-free rate.

4.2. Simulated measures of serial correlation in returns

Strong negative serial correlation in long-horizon returns can be induced by a slow mean reverting temporary component in asset prices, but as mentioned in the introduction the economic rationale for this mean reversion in prices is not unique: both inefficient and efficient market explanations can be made consistent with this observation. For the proponents of an inefficient market, high (low) dividend price ratios announce high (low) returns because stock prices are irrationally low (high)

[16] Increasing β over unity as suggested by Kocherlakota (1990) leaves the equity premium mean unaffected. For example, setting β to 1.05 and γ to 5 lowers the means of the equity return and of the risk-free rate to 4.89% and 4.37% respectively, leaving the equity premium unchanged at about 0.50% compared with the value obtained with β equal to 0.97 and the same γ.

[17] Cecchetti et al. (1993) did not use these criteria in assessing their model. We arrive at this conclusion by using the consumption and dividend parameter values reported in their paper and trying the various values of the preference parameters reported in their Table 3.

compared to their fundamental value. Mean reversion in prices is created by the intervention, at some point, of arbitrators to eliminate this swing away from the fundamental value. The efficient market explanation (Fama and French, 1988a,b) argues that if equilibrium expected returns are highly correlated but mean reverting and if shocks to expected returns are independent of the shocks to rational forecasts of dividends, a shock to expected returns has no long term effect on expected prices and must therefore be compensated by an opposite movement in the current price. The mean reversion in prices is then implied by the mean reversion in expected returns. One way to distinguish between the two theories is to propose an equilibrium model that imposes restrictions on the evolution of expected returns. In this section, we show that the model presented in Section 2 produces some negative autocorrelation in real returns, but fails to do so for excess returns, another piece of evidence in line with our results on the unconditional moments.

Tables 4 and 5 present the results of our Monte-Carlo experiments for real returns and excess returns respectively. Given a randomly drawn vector of $N(0,1)$ errors ϵ^d_{t+1} and a randomly drawn vector of $S_{0,t}$, $S_{1,t}$, and $S_{2,t}$ according to the transition probabilities estimated in Section 3, we generate series of equity and excess returns according to formulas (20) and (21) with the estimates obtained in Section 3 for the α, ω and ρ parameters. We replicate the procedure a 1,000 times and compute each time the variance ratios at lags 2 to 10 and the 1 to 10 multiperiod returns regression coefficients as defined at the top of Tables 4 and 5. We therefore obtain the respective distributions of variance ratios and regression coefficients at various lags. For the length of the series, we choose 116 observations (the number of observations for the actual returns) to generate the small sample distributions and 1,160 observations for the large sample ones to account for small sample bias. We report the medians of the distributions for the variance ratios and the regression coefficients both for small samples (SS) and large samples (LS) as well as, in the case of the small sample distribution, the percentage (%) of the distribution below the actuals. These percentages are to be interpreted as p-values for the hypothesis that the actuals are produced by the model. The closer they are to the 50% line, the more support for the hypothesis. As in Section 4.1, we have selected the values 1.5, 5 and 10 for γ and 0.97 for β.

For the real returns (Table 4), the best results among these three values of γ are obtained for $\gamma = 5$. The actuals all lie close to the 40% line of the small sample variance ratio distributions, while there is still evidence of some negative autocorrelation in the large sample medians. [18] This evidence is confirmed by the

[18] It can be argued that the negative serial correlation exhibited by the actual returns is purely due to small sample bias and should disappear in large samples. Our purpose in this simulation exercise is to assess first how likely it is for the actuals to have been produced by such a model (this is the % of the distribution below the actuals) and second if the model can produce negative correlation in large samples, which is essential for the equilibrium explanation of mean reversion in asset prices.

Table 4
Simulated measures of serial correlation: Real returns

Median of distribution of variance ratios of real returns for model calibrated to the joint consumption-dividend three-state Markov model										
k	Actual	$\gamma = 1.5$			$\gamma = 5$			$\gamma = 10$		
		SS	%	LS	SS	%	LS	SS	%	LS
2	1.0256	0.9894	0.61	0.9966	0.9687	0.69	0.9758	0.9667	0.71	0.9736
3	0.9044	0.9744	0.34	0.9925	0.9435	0.39	0.9595	0.9476	0.40	0.9558
4	0.8785	0.9576	0.33	0.9916	0.9199	0.40	0.9478	0.9313	0.39	0.9492
5	0.8572	0.9433	0.34	0.9899	0.9009	0.42	0.9387	0.9212	0.39	0.9471
6	0.7941	0.9292	0.29	0.9868	0.8833	0.35	0.9312	0.9113	0.32	0.9491
7	0.7625	0.9142	0.28	0.9836	0.8647	0.34	0.9286	0.9054	0.29	0.9553
8	0.7781	0.8946	0.35	0.9814	0.8435	0.39	0.9242	0.8975	0.34	0.9604
9	0.7646	0.8784	0.35	0.9785	0.8312	0.40	0.9202	0.8822	0.34	0.9682
10	0.7395	0.8665	0.34	0.9745	0.8173	0.39	0.9154	0.8735	0.32	0.9743

Median of distribution of regression coefficients of real returns for model calibrated to the joint consumption-dividend three-state Markov model										
k	Actual	$\gamma = 1.5$			$\gamma = 5$			$\gamma = 10$		
		SS	%	LS	SS	%	LS	SS	%	LS
1	0.0188	−0.0123	0.59	−0.0031	−0.0326	0.68	−0.0241	−0.0330	0.69	−0.0267
2	−0.1539	−0.0316	0.16	−0.0040	−0.0497	0.21	−0.0310	−0.0351	0.18	−0.0210
3	−0.1432	−0.0458	0.26	−0.0087	−0.0604	0.31	−0.0306	−0.0292	0.23	−0.0324
4	−0.1376	−0.0639	0.34	−0.0095	−0.0682	0.36	−0.0277	−0.0300	0.28	0.0153
5	−0.1601	−0.0767	0.31	−0.0102	−0.0862	0.33	−0.0243	−0.0304	0.24	0.0303
6	−0.0957	−0.0925	0.49	−0.0101	−0.1040	0.52	−0.0240	−0.0242	0.37	0.0423
7	−0.1037	−0.1038	0.50	−0.0121	−0.1207	0.54	−0.0232	−0.0261	0.38	0.0526
8	−0.1982	−0.1124	0.37	−0.0117	−0.1364	0.40	−0.0236	−0.0346	0.27	0.0605
9	−0.2792	−0.1243	0.26	−0.0114	−0.1465	0.30	−0.0251	−0.0528	0.19	0.0693
10	−0.3829	−0.1391	0.17	−0.0130	−0.1547	0.18	−0.0265	−0.0713	0.11	0.0749

Table 5
Simulated measures of serial correlation: Excess returns

Median of distribution of variance ratios of excess returns for model calibrated to the joint consumption-dividend three-state Markov model

k	Actual	$\gamma = 1.5$			$\gamma = 5$			$\gamma = 10$		
		SS	%	LS	SS	%	LS	SS	%	LS
2	1.0957	0.9975	0.81	1.0019	1.0028	0.77	1.0114	1.0144	0.77	1.0270
3	1.0012	0.9822	0.55	1.0027	0.9955	0.51	1.0141	1.0229	0.44	1.0459
4	0.9849	0.9667	0.53	1.0027	0.9826	0.50	1.0188	1.0319	0.41	1.0636
5	0.9361	0.9473	0.48	1.0037	0.9653	0.44	1.0220	1.0290	0.34	1.0745
6	0.8509	0.9396	0.38	1.0005	0.9531	0.33	1.0242	1.0217	0.24	1.0847
7	0.7906	0.9268	0.32	1.0015	0.9417	0.28	1.0282	1.0229	0.19	1.0934
8	0.7795	0.9149	0.33	0.9996	0.9354	0.29	1.0319	1.0129	0.21	1.0991
9	0.7710	0.9053	0.35	0.9973	0.9181	0.31	1.0344	1.0083	0.22	1.1054
10	0.7828	0.8903	0.39	0.9979	0.9109	0.36	1.0344	0.9966	0.26	1.1104

Median of distribution of regression coefficients of excess returns for model calibrated to the joint consumption-dividend three-state Markov model

k	Actual	$\gamma = 1.5$			$\gamma = 5$			$\gamma = 10$		
		SS	%	LS	SS	%	LS	SS	%	LS
1	0.0871	−0.0034	0.80	0.0015	0.0000	0.75	0.0114	0.0127	0.75	0.0270
2	−0.1142	−0.0224	0.25	0.0004	−0.0129	0.23	0.0121	0.0144	0.15	0.0358
3	−0.1668	−0.0341	0.20	0.0000	−0.0376	0.18	0.0105	0.0040	0.12	0.0363
4	−0.2128	−0.0513	0.17	0.0000	−0.0434	0.17	0.0093	−0.0080	0.11	0.0361
5	−0.1680	−0.0582	0.31	−0.0033	−0.0577	0.30	0.0105	−0.0273	0.20	0.0357
6	0.0082	−0.0767	0.66	−0.0062	−0.0784	0.65	0.0091	−0.0357	0.58	0.0363
7	0.1133	−0.0886	0.81	−0.0085	−0.0844	0.80	0.0103	−0.0503	0.76	0.0359
8	0.1025	−0.1139	0.81	−0.0093	−0.1010	0.79	0.0120	−0.0627	0.74	0.0348
9	0.0841	−0.1263	0.79	−0.0112	−0.1061	0.77	0.0095	−0.0702	0.74	0.0356
10	0.0606	−0.1440	0.78	−0.0150	−0.1158	0.76	0.0091	−0.0847	0.72	0.0320

regression coefficients, where for $\gamma = 5$ the only actual regression coefficient below the 20% line is at lag 10.

However, for the excess returns (Table 5), even if the actuals of the variance ratios are all above the 30% line of the small sample distributions for $\gamma = 1.5$, all evidence of negative autocorrelation disappears in large samples for all values of γ. Moreover, the actual values of the regression coefficients all lie close to the 20% or 80% lines of the small sample distributions for all values of γ.

We therefore conclude that the model can produce some negative autocorrelation in real returns, although not quite as much as in the actual data, but fails to produce any in excess returns.

4.3. Simulated statistics for the regressions of multiperiod returns on dividend yields

Given the fact that univariate tests on returns have little power, Fama and French (1988b) proposed to regress one- to four-year returns on the dividend-price ratio. They found statistical evidence of a larger predictable component for long-horizon returns. The actuals in Table 6 confirm their results for the period 1871–1985 [19] and show that the regression coefficients increase slightly less than in proportion with the horizon for both the real returns and the excess returns. As Fama and French (1988b) explain, this means, since multiperiod returns are cumulative sums of one-period returns, that the dividend-price ratio does not predict as much variation in the distant one-period expected returns, an indication of slow mean reversion in short term expected returns. Because of this slow mean reversion, short term expected returns are persistent, and the variance of multiperiod expected returns grows more than in proportion with the return horizon. However, the variance of the regression residuals grows much less with the return horizon, as shown in Table 1 (a standard deviation of 0.27 for two-year returns compared to 0.34 for four-year returns). The latter fact indicates that the residuals from the one-year regressions must on average be negatively autocorrelated. This explains why the forecasting power increases with the horizon.

We proceed as in Section 4.2 to generate the return series but we note that if we assume that the agent knows the state at time t, the model gives us only three values for the dividend-price ratio. To obtain a continuous variable for the price-dividend ratio, we therefore assume that the state is not directly observable [20] and allow the agent to make an optimal inference about the probabilities of states 0, 1, and 2 at time t given his information up to time t and, of course, the values of the parameters of the model. These inferred probabilities are precisely what we

[19] They study the period 1926–1985 and various sub-periods.

[20] This assumption changes, of course, the model we used to generate the measures of serial correlation. The implications for serial correlation in returns of this new assumption regarding the information available to the agent should be investigated.

Table 6
Simulated statistics on the forecastability of future returns by current dividend yields regression: $R_{t,t+k} = \alpha + \beta(D_t / P_t) + u_{t,t+k}$

Median of distribution of various regression statistics: Real returns

	Actual			$\gamma = 1.5$ $\beta = 0.97$			$\gamma = 7$ $\beta = 0.97$			$\gamma = 10$ $\beta = 0.97$		
k	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2
1	2.87	1.91	0.03	0.85	0.21	0.004	1.40	1.71	0.03	1.39	2.83	0.07
2	5.31	2.48	0.05	1.69	0.30	0.006	2.53	2.18	0.04	2.61	3.80	0.11
3	7.05	2.89	0.07	2.05	0.30	0.009	3.52	2.40	0.05	3.67	4.52	0.16
4	10.10	3.72	0.11	2.47	0.33	0.012	4.33	2.63	0.06	4.62	4.96	0.18
5	13.59	4.68	0.17	3.19	0.34	0.015	5.05	2.76	0.07	5.40	5.32	0.21
	Actual			Percentage of distribution below the actual values								
1	2.87	1.91	0.03	0.72	0.95	0.94	0.91	0.60	0.60	0.94	0.16	0.16
2	5.31	2.48	0.05	0.71	0.95	0.94	0.92	0.60	0.60	0.96	0.15	0.15
3	7.05	2.89	0.07	0.69	0.94	0.92	0.92	0.61	0.61	0.96	0.18	0.18
4	10.10	3.72	0.11	0.72	0.96	0.94	0.96	0.70	0.70	0.98	0.26	0.26
5	13.59	4.68	0.17	0.74	0.97	0.96	0.98	0.79	0.79	0.99	0.39	0.39

Median of distribution of various regression statistics: Excess returns

	Actual			$\gamma = 1.5$ $\beta = 0.97$			$\gamma = 7$ $\beta = 0.97$			$\gamma = 10$ $\beta = 0.97$		
k	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2
1	3.46	2.35	0.05	−0.12	−0.03	0.003	−0.36	−0.42	0.005	−0.21	−0.44	0.005
2	6.45	3.01	0.08	−0.34	−0.06	0.006	−0.61	−0.51	0.009	−0.39	−0.55	0.008
3	8.29	3.36	0.09	−0.22	−0.03	0.009	−0.88	−0.58	0.011	−0.51	−0.58	0.011
4	11.16	3.99	0.13	−0.21	−0.03	0.011	−0.95	−0.60	0.014	−0.58	−0.60	0.014
5	14.51	4.92	0.18	−0.55	−0.05	0.015	−1.01	−0.57	0.017	−0.66	−0.57	0.018
	Actual			Percentage of distribution below the actual values								
1	3.46	2.35	0.05	0.85	0.98	0.97	0.997	0.99	0.97	0.998	0.99	0.96
2	6.45	3.01	0.08	0.84	0.98	0.96	0.996	0.99	0.95	-	0.99	0.96
3	8.29	3.36	0.09	0.79	0.98	0.96	0.996	0.99	0.94	-	0.99	0.95
4	11.16	3.99	0.13	0.79	0.98	0.96	0.996	0.99	0.96	-	0.99	0.95
5	14.51	4.92	0.18	0.80	0.99	0.98	0.997	0.99	0.97	-	0.99	0.97

called in Section 3 the filter probabilities. We therefore obtain the price-dividend ratio by weighing the three values of $\rho(S_t)$ by these filter probabilities.

As shown in the upper part of Table 6, the model produces the same patterns as in the data for values of γ between 7 and 10 in the case of real returns, except that the values of the regression coefficient medians are too low: all the coefficients are positive, growing with the return horizon, as does the forecasting power. The actuals stand at about 40% of the distribution of the t and R^2 statistics. However, the model fails totally when it comes to excess returns. The coefficients are negative and totally insignificant, which translates into R^2 close to zero.

To summarize our assessment of the model, we can say that the model fails in all dimensions involving the excess returns. However, because we relaxed the restriction about the equality of consumption and dividends, we were able to reproduce, although not fully and with γ values between 5 and 10, the features associated with the real equity returns. Given that our characterization of risk is as close to the data as possible, two main avenues can be explored to try and explain the failures of our asset pricing model. The first is to investigate different specifications for preferences, along with our risk specification. This is done with some success in Bonomo and Garcia (1993). [21] Another is to abandon the assumptions of complete markets and of a representative agent.

The solution to the equity premium puzzle mentioned by Mehra and Prescott (1985) was to introduce heterogeneity between consumers. Mankiw (1986) and Weil (1990) propose models showing that if the agents bear some idiosyncratic undiversifiable risk, their consumption will be more risky, justifying a high equity premium. Also, liquidity constraints could prevent some consumers from participating in financial markets. The consumption process that will rationalize the observed returns will therefore differ from aggregate consumption. One can therefore ask the following question: How risky (variable) should the consumption of a representative agent holding the stock and the risk-free asset be to justify the observed equity and safe asset returns? We will give a numerical answer to this question in the next section within the framework of our parametric model.

5. Inferring the consumption parameters that rationalize observed returns

Our goal in this section is to assess the empirical plausibility of the model in terms of mean and variance of individual consumption growth inferred from asset market data. This is similar in spirit to Hansen and Jagannathan (1991), since given our choice of power utility function, the means and variances of the consumption growth can be translated into intertemporal marginal rates of substi-

[21] In this paper, the authors show that both the first and second moments of the equity premium and the risk-free rate can be matched by endowing the agents in the economy with disappointment aversion preferences.

tution (IMRS) by raising them to the power γ and discounting them with β. Their goal, however, is more general and consists in deriving non-parametrically admissible regions for the means and variances of IMRS in order to test the validity of many classes of models. Of course, by not imposing a parametric structure, they only derive bounds for the IMRS and we trade-off this generality for getting estimates of the means and variances of the consumption process.

To infer the consumption parameters from actual returns, we start with the theoretical formulas given by the model for the returns. Taking the natural logarithm of the return formulas (20) and (21) for the risk-free asset and the equity respectively (assuming three states) and using the dividend process assumed in Eq. (8), we obtain the following system:

$$\begin{aligned}
&\ln R_t^e - (\ln D_{t+1} - \ln D_t) = \ln[\rho(S_{t+1}) + 1] - \ln[\rho(S_t)] + v_t^e,\\
&\ln R_t^f = -\ln \phi(S_t) + v_t^f,\\
&\ln D_t - \ln D_{t-1} = \alpha_0^d + \alpha_1^d S_{1,t} + \alpha_2^d S_{2,t} + \left(w_0^d + \omega_1^d S_{1,t} + \omega_2^d S_{2,t}\right)\epsilon_t^d, \qquad (22)
\end{aligned}$$

where normal mean-zero measurement error terms v_t^e and v_t^f, with variances σ_d^2 and σ_f^2 respectively, have been added to the equity return and risk-free return equations respectively. This system can be estimated with a trivariate version of the Markov switching model algorithm, as follows:

$$\begin{aligned}
&r_t^e = a + bS_{1,t} + cS_{2,t} + \ln(\exp(a) + 1)P(S_t,0)\\
&\qquad + \ln(\exp(a+b) + 1)P(S_t,1)\\
&+ \ln(\exp(a+c) + 1)P(S_t,2) + v_t^e,\\
&r_t^f = d + eS_{1,t} + fS_{2,t} + v_t^f,\\
&d_t - d_{t-1} = \alpha_0^d + \alpha_1^d S_{1,t} + \alpha_2^d S_{2,t} + \left(\omega_0^d + \omega_1^d S_{1,t} + \omega_2^d S_{2,t}\right)\epsilon_t^d. \qquad (23)
\end{aligned}$$

We do not impose any restrictions on the covariance matrix of the errors, which leaves us with 23 parameters to estimate. The estimation results are reported in Table 7.

Note first that the estimates of the transition probability parameters are close to what we obtained with the estimation of the joint consumption-dividend process, except that state 2 (labelled state 1 before) is more persistent (0.949 instead of 0.854). The other transition probabilities are almost identical. This is a good feature since we are estimating the transition probability matrix of the same Markov variable. We also observe that the means of the dividend process are close to zero as in our estimation of the joint consumption-dividend process, but that the standard deviation parameter in state 1 (labelled state 0 before) is much higher. This is probably due to the fact that we do not allow the variances in the other two equations to change with the state as we did before when estimating the joint process with consumption.

The estimate of -2.669 for the coefficient a translates into a price-dividend ratio of 14.43, but the estimates of b and c are not significantly different from zero, so that the dividend-price ratio does not vary significantly between states.

Table 7
Estimation of the trivariate Markov system with actual returns and dividend growth

Parameter	Coefficient estimate	Standard error
a	−2.669	(0.221)
b	−0.058	(0.169)
c	−0.005	(0.210)
d	0.012	(0.007)
e	−0.146	(0.016)
f	0.032	(0.009)
α_0^d	−0.005	(0.008)
α_1^d	−0.035	(0.07)
α_2^d	0.007	(0.02)
ω_0^d	−0.048	(0.008)
ω_1^d	0.245	(0.046)
ω_2^d	0.202	(0.017)
σ^e	0.133	(0.009)
σ^f	0.043	(0.003)
ρ^{ef}	−0.009	(0.095)
ρ^{de}	−0.118	(0.09)
ρ^{df}	−0.235	(0.09)
p_{01}	0.056	(0.00)
p_{02}	0.000	(0.00)
p_{11}	0.546	(0.161)
p_{12}	0.353	(0.159)
p_{21}	0.032	(0.023)
p_{22}	0.949	(0.027)
L	641.83	

Note: To identify the estimated parameters, refer to the system of Eq. (23).

For the risk-free asset, the prices in state 0, 1 and 2 are respectively 0.988, 1.14 and 0.644. In state 1, the agent is ready to pay more than the asset is worth and incur a negative real return.

Finally, as it should be, the correlation coefficient parameters are all small and not significantly different from zero except ρ^{df}, the correlation coefficient between ϵ_t^d and v_t^f.

Given these estimates, we can solve the set of six non-linear equations defined by Eq. (17) and Eq. (18) to recover the consumption parameters, although we need to assume some values for γ, β and ρ^{cd}.

Table 8 shows the implied parameters for the consumption process for various γ values, when the utility discount rate is 0.97 and the correlation between consumption and dividend innovations is 0.85. [22] We see that all the conditional mean and variance parameters, as well as the unconditional mean and standard

[22] This value has been chosen since we expect the correlation with the innovations in dividend growth to be higher for consumption growth of an individual stockholder than for the aggregate consumption growth, for which we find a value of 0.506.

Table 8
Implied parameters for the individual consumption process

γ	2	5	10
α_0	0.2149	0.0860	0.0430
α_1	0.2506	0.1003	0.0501
α_2	−0.1911	−0.0763	−0.0382
ω_0	−0.4630	−0.1852	−0.0926
ω_1	1.246	0.4985	0.2492
ω_2	0.5667	0.2267	0.1133
Mean [a]	0.1922	0.05206	0.0230
S.d. [b]	1.109	0.4859	0.3121

Notes: For the results above we set $\beta = 0.97$ and $\rho_{cd} = 0.85$.
[a] Mean of the consumption growth.
[b] Standard deviation of the consumption growth.

deviation of the consumption growth, decrease in magnitude when γ increases. This result conforms to intuition since a higher standard deviation and a larger difference between state means are needed with a lower γ to rationalize the characteristics of equity and riskless asset returns, and especially a large equity premium. The more risk averse a consumer is, the less variability is required in her consumption (which covaries with the equity return) to explain her preference for the riskless asset over the equity. [23]

The reason for the decreasing relation between the average consumption growth and γ is also clear. The lower the elasticity of intertemporal substitution, the lower the consumption growth necessary to generate the marginal rate of substitution which rationalizes a given riskless return. Then, since in the particular utility function we use, γ represents both the coefficient of relative risk aversion and the inverse of the elasticity of intertemporal substitution, we obtain a positive correlation between the mean and the standard deviation of consumption growth in the results of Table 8. Note that when γ is equal to 10, the mean of the consumption growth is of the same order of magnitude as the actual mean of aggregate consumption growth, but the standard deviation is much higher. This result illustrates why the model could not succeed in solving the equity premium or the risk-free puzzles: much more variability than present in the aggregate consumption series is needed to match the actual equity premium and the risk-free rate. Moreover, the results obtained show a strong heteroskedasticity in the consumption process, at least between state 1 and the other states. Finally, the value of 0.31

[23] The same intuition can explain why the correlation between consumption and dividend innovations decreases, ceteris paribus, when γ increases.

obtained for the standard deviation of the consumption process seems too high even for a representative stockholder. [24]

6. Conclusion

This paper started with some stylized facts regarding real and excess returns. Our conclusion is that an equilibrium asset pricing model based on an exchange economy – the particular version presented or an improved version based on existing refinements – can replicate roughly the features associated with real returns, but is totally incapable of replicating the excess returns characteristics. The equity premium and the risk-free rate puzzles are precisely a version of this result, but we confirmed this evidence using other statistics such as the negative autocorrelation of returns and the forecasting power of the dividend-price ratio for multiperiod returns.

Given this evidence, one can follow two routes: explore further the preference specification as in Bonomo and Garcia (1993) or follow Mehra and Prescott's advice in searching for an incomplete market and heterogeneous consumers explanation (Mehra and Prescott, 1985). [25] In Section 5, we illustrated how an intertemporal asset pricing model could still be used to guide our search in trying to characterize some representative consumption process that will rationalize the returns observed on the market. These benchmark figures can be compared to figures resulting from future micro-level studies aimed at identifying the unconstrained participants in the stock market if, for example, one is ready to invoke liquidity constraints as a source of friction in the economy. These micro-studies could also help the model building exercise and especially the choice of a good utility function. These are important issues we intend to address in future research.

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[24] For the standard deviation of individual income growth, MaCurdy (1982) reports a value of 0.25.

[25] Another approach for testing intertemporal asset pricing models is to avoid reference to consumption (Campbell, 1993).

Appendix A. Data Sources

- Nominal dividends and stock prices: Campbell and Shiller (1987) data set.
- Price index: CPI
  1871–1926: Wilson and Jones (1987)
- 1930–1985: Ibbotson and Sinquefield (1988)
- Nominal interest rate: constructed from four different sources as in Cecchetti et al. (1990, data appendix)
- Consumption:
  1889–1928: Kendrick Consumption series (Balke and Gordon, 1986)
  1929–1985: NIPD Accounts
- Population:
  1869–1938: Historical Statistics of the United States (Series A7 for 1869–1928, Series A6 1929–1938)
  1938–1985: Economic Report of the President (1989), Table B-31.

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