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Tail risk and asset prices in the short-term

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Abstract

We combine high-frequency stock returns with risk-neutralization to extract the daily common component of tail risks perceived by investors in the cross-section of firms. Our tail risk measure significantly predicts the equity premium and variance risk premium at short horizons. Furthermore, a long–short portfolio built by sorting stocks on their recent exposure to tail risk generates abnormal returns with respect to standard factor models. Incorporating investors' preferences via risk-neutralization is fundamental to our findings: the predictive power of the physical tail risk is weaker and generally subsumed by its risk-neutral counterpart.

Keywords left-tail risk, return predictability, factor models, risk-neutralization, high-frequency data.
JEL classifications C58, G12, G17.

1. Introduction

Left-tail risk is a pervasive feature of financial markets. As such, a large body of work has investigated its role in determining asset prices. Taken together, the empirical evidence indicates that compensation required by investors for bearing tail risk is fundamental to explain aggregate market risk premia and the cross-section of stock returns at relatively low frequencies (monthly or longer).[1] This evidence is based on a number of different tail measures. In

[1] The risk of extreme left tail shocks to fundamentals has also been an important ingredient in equilibrium models to help rationalize asset pricing stylized facts that are puzzling from the perspective of traditional macro-finance models (Barro 2006; Gabaix 2012; Wachter 2013).

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particular, information can be extracted either from stock prices (e.g., Bali, Demirtas, and Levy 2009; Kelly and Jiang 2014), reflecting risk under the physical or statistical measure under which prices are observed, or from option prices (e.g., Andersen, Fusari, and Todorov 2015; Bollerslev, Todorov, and Xu 2015), capturing tail risk under the risk-neutral measure incorporating investors' preferences.

In this article, we propose a new tail measure available at a daily frequency, which allows us to investigate the short-term effects of tail risk on asset prices. We first estimate the common tail risk component of a large cross-section of intra-day stock returns on day t, $\lambda_t^{\mathbb{P}}$, using the Hill (1975) power law estimator. This essentially adapts the tail index by Kelly and Jiang (2014) to a high-frequency environment. Then, we introduce a novel version of the Hill estimator, $\lambda_t^{\mathbb{Q}}$, that relies on risk-neutralized returns. More specifically, we apply a nonparametric adjustment to the cross-section of stock returns on day t where "bad" states of nature, represented by states of high marginal utility, are overweighted to reflect investors' compensation for risk. The dynamics of the physical and risk-neutral Hill estimators differ substantially as compensation for risk varies over time.

Our approach overcomes two main challenges. First, extreme events are infrequently observed by definition. This limits the information available from the time series of a single asset such as the market index. Second, option maturities are relatively long compared to intra-daily events, which makes it difficult to measure the tail risk specific to day t using option prices.[2] By using high-frequency data on one of the largest cross-sections of stock returns available, we are able to extract information about the level of tail risk at day t from the individual extreme events experienced by different stocks. Furthermore, our risk-neutralization allows to obtain a tail measure incorporating the economic valuation of tail risks by investors, which otherwise would only be possible using option prices (see, e.g., Aït-Sahalia and Lo 2000).

Our objective is to analyze how risk premia at future short horizons respond to tail risk embedded in the cross-section of intra-day stock returns on day t. Our central hypothesis is that investors are tail risk averse, demanding higher risk premia when both the level of tail risk and their aversion to it increase. Both components—risk and risk prices—are naturally incorporated in $\lambda_t^{\mathbb{Q}}$, which takes into account investors' preferences. In contrast, the physical measure of tail risk, $\lambda_t^{\mathbb{P}}$, can be high (low) while tail risk aversion is low (high), particularly when extreme negative returns occur in states of low (high) marginal utility. Consequently, if our risk-neutralization procedure is effective, we should expect $\lambda_t^{\mathbb{Q}}$ to exhibit stronger predictive power for risk premia than $\lambda_t^{\mathbb{P}}$.[3]

In our empirical analysis, we consider each of our tail risk measures ($\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$) in order to assess the information content of investors' economic valuation of tail risk. Interestingly, both tail measures are weakly correlated with volatility-based measures. This is because the Hill estimator captures the thickness of the left tail after taking into account the effect of volatility.[4] Moreover, $\lambda_t^{\mathbb{Q}}$ is usually greater than $\lambda_t^{\mathbb{P}}$, which reflects the fact that investors are averse to downside risk. However, this difference varies over time and is particularly large during financial crises. In other words, investors are especially afraid of extreme negative events during periods of market distress.

[2] Zero days-to-expiration (0DTE) option contracts are an exception (see, e.g., Bandi, Fusari and Renò 2023; Almeida, Freire, and Hizmeri 2024). However, as of now, the sample for which these options are available at a daily frequency (from May 2022 onwards) is still rather limited.

[3] In our risk-neutralization, negative stock returns observed during intra-daily intervals with negative (positive) systematic factor returns are over (down) weighted to reflect high (low) marginal utility.

[4] Gormsen and Jensen (2024) document a similar pattern for longer horizons: tail probabilities defined based on a time-varying threshold that depends on the conditional volatility are mostly pro-cyclical.

We start by examining the short-term relation between the tail measures and the equity premium with 1 day, 1 week, and 1 month ahead daily predictive regressions. We find that the risk-neutral tail risk positively predicts excess market returns, with statistical significance for all horizons considered. This is consistent with the idea that investors are averse to tail risk, such that they require higher returns to hold the market when tail risk and their aversion to it increase. In contrast, $\lambda_t^{\mathbb{P}}$ only has predictive power for the 1-month horizon. When both measures are included, $\lambda_t^{\mathbb{Q}}$ remains a significant predictor of market returns, while $\lambda_t^{\mathbb{P}}$ becomes insignificant. We also consider a fully out-of-sample predictive exercise. Again, risk-neutral tail risk consistently predicts excess market returns across all horizons, while its physical counterpart only contains information about the 1-month horizon or longer. Importantly, exploiting this predictability in a portfolio choice problem generates substantial utility gains for a mean–variance investor.

The predictive power of $\lambda_t^{\mathbb{Q}}$ for the equity premium is robust to controlling for several alternative predictors, such as realized risk measures obtained from high-frequency returns and option-implied tail risk measures. In fact, the risk-neutral tail risk is the strongest predictor of future excess market returns among the controls. The only exception is the variance risk premium (Bollerslev, Tauchen, and Zhou 2009), which provides even stronger explanatory power and drives out the effect of $\lambda_t^{\mathbb{Q}}$ at the 1-day horizon. This indicates that the variance risk premium helps explain variation in future excess market returns not only at lower frequencies, as previously shown in the literature, but also in the short-term.

The variance risk premium should, nonetheless, be largely explained by investors' aversion to downside risk (Bollerslev, Todorov, and Xu 2015). Accordingly, our $\lambda_t^{\mathbb{Q}}$ measure strongly predicts the variance risk premium with a positive sign across all horizons considered, meaning that investors demand higher compensation for bearing market variance risk when risk-neutral tail risk is higher. The physical tail risk is also useful for predicting the variance risk premium. However, when we include both tail measures in the predictive regressions, only $\lambda_t^{\mathbb{Q}}$ remains significant. Such predictability also survives the inclusion of and is the strongest among several controls, such as realized and option-implied measures of continuous and jump variation. In other words, the perception of tail risk extracted from the cross-section of high-frequency returns is highly informative about the future variance risk premium in the short-term.

While $\lambda_t^{\mathbb{Q}}$ predicts aggregate market risk premia, our main contribution is to show that short-term tail risk is priced in the cross-section of stocks. To do so, for each of our tail measures, we build a long–short portfolio by sorting stocks each month on their recent exposure to the measure, based on contemporaneous daily regressions.[5] The tail factor constructed from $\lambda_t^{\mathbb{Q}}$ generates statistically significant average returns that cannot be explained by standard factor models, where stocks with high exposure to tail risk have high hedging capacity and are thus highly priced, yielding subsequent low returns. In contrast, $\lambda_t^{\mathbb{P}}$ leads to insignificant spreads in returns, that is, only the short-term exposure to tail risk as perceived by investors explains differences in expected returns across stocks. This evidence is further confirmed with Fama–MacBeth regressions, including a large set of relevant controls. Finally, given the importance of the variance risk premium in predicting the equity premium, we use conditional double sorts to investigate whether controlling for it can account for the cross-sectional effects of $\lambda_t^{\mathbb{Q}}$. We find that sorting on $\lambda_t^{\mathbb{Q}}$ yields highly significant average returns

[5] In contrast to papers focusing on high-frequency factors and continuous-time asset pricing models (e.g., Bollerslev, Li, and Todorov 2016; Aït-Sahalia, Kalnina, and Xiu 2020; Pelger 2020), we build a factor at the usual monthly horizon by sorting stocks on their exposure to a daily tail measure obtained from high-frequency returns.

and alphas regardless of the variance risk premium quintile, reinforcing the complementarity between the two measures.

We conduct a comprehensive battery of robustness tests to show that our conclusions remain under alternative modeling choices, including: different thresholds to define the left tail in the return distribution, different hyperparameters related to our risk-neutralization procedure, excluding overnight stock returns, and accounting for cross-sectional and time-of-day patterns in volatility when calculating tail risk. The remainder of the article is organized as follows. After a brief discussion of the related literature, Section 2 describes the methodology to construct our tail measures. Section 3 presents the data and the estimated tail measures, while Section 4 contains our empirical analysis. Section 5 concludes the article. Lastly, Appendices A and B contain the description of standardized measures and variable definitions, respectively.

1.1 Related literature

Our article is mainly related to the evolving literature on left-tail risk and investors' compensation for such risk. Bollerslev and Todorov (2011), Bollerslev, Todorov, and Xu (2015), and Andersen, Fusari, and Todorov (2015, 2017) provide evidence that tail risk is an important determinant of the equity and variance risk premia using option-implied tail measures. Extracting information from stock prices, Bali, Demirtas, and Levy (2009), Kelly and Jiang (2014), and Almeida et al. (2017) show that tail risk predicts future market returns and macroeconomic activity. Computing tail risk at the firm level, Ang, Chen, and Xing (2006), Bali, Cakici, and Whitelaw (2014), Chabi-Yo, Ruenzi, and Weigert (2018), Atilgan et al. (2020), and Bollerslev, Patton, and Quaedvlieg (2022) document significant cross-sectional relations between tail risk and future stock returns. Bégin, Dorion, and Gauthier (2020) highlight the importance of tail risk in the pricing of idiosyncratic risk, and international evidence is provided by Andersen, Fusari, and Todorov (2020), Andersen, Todorov, and Ubukata (2021), and Freire (2021). We contribute by proposing a novel method to estimate the tail risk of day t, exploiting rich cross-sectional information from high-frequency stock returns, and documenting new short-term return predictability for the aggregate market and the cross-section, with particular focus on the role of incorporating investors' preferences towards tail risk.

The closest work to ours is by Kelly and Jiang (2014), who propose the Hill estimator to compute the common tail risk component of a cross-section of stocks at a monthly frequency.[6] We adapt their estimator to the daily frequency using intra-day returns and develop a new risk-neutral version, where we show that only the latter helps explain the equity premium, the variance risk premium, and the cross-section of returns. Also closely related, Weller (2019) exploits the cross-section of intra-day bid and ask quotes to estimate tail risk and investigate predictability of jumps in high-frequency. We rely on the cross-section of intra-day returns and risk-neutralization to obtain the perception of tail risk and assess its role in determining risk premia over short horizons. Finally, Almeida et al. (2024) use high-frequency market returns to construct an expected shortfall premium that predicts 1-day-ahead market risk premia. We extract information about left-tail risk from intra-day extreme events across many stocks using the Hill estimator. We show that it predicts market risk premia at different short horizons and investigate how short-term exposure to tail risk is priced in the cross-section.

[6] In a very recent paper, Andersen, Ding, and Todorov (2024) also study tail risk from high-frequency stock returns, but focus on disentangling systematic and idiosyncratic jump risk, and only under the physical measure.

2. Methodology

In this section, we describe our approach to estimate left-tail risk at a daily frequency. Using a cross-section of intra-day stock returns, we first extract information about the common component of the tail risks of individual firms using the Hill estimator. Then, we introduce a new version of the Hill estimator that relies on risk-neutralized stock returns, thus taking into account the investors' perception of risk in the estimation.

2.1 Hill estimator

Extreme events in financial markets are rare by definition. This makes it challenging to construct an aggregate measure of tail risk relying on a single asset such as the market index, since informative observations for the tail are infrequent. To overcome this issue, we follow Kelly and Jiang (2014) by adopting a panel estimation approach capturing common tail behavior in the cross-section of individual stock returns. The identifying assumption is that the dynamics of the tail distributions of the firms are similar, so that extreme events in the cross-section allow us to extract the common component of their tail risk at each point in time.

More specifically, we assume that the left tail of the return distribution of asset i follows a power law structure.[7] That is, its day t conditional left-tail distribution, defined as the set of extreme returns below some negative threshold u_t, obeys the following:

$$P(R^i_{t+1} < r | R^i_{t+1} < u_t \text{ and } \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_i/\lambda_t}, \tag{1}$$

where $r < u_t < 0$ and $\mathcal{F}_t$ is the conditioning information set.[8] The parameter a_i/λ_t is the tail exponent, which determines the shape of the tail distribution of asset i. The constant a_i may be different across assets in the cross-section, implying that they can have different levels of tail risk. However, their dynamics are driven by a common time-varying component, λ_t. The higher the λ_t, the thicker the stock returns' left tails and the higher the probabilities of extreme negative returns in the cross-section. Therefore, we refer to λ_t as our measure of aggregate tail risk.[9]

For each day t in our sample, we estimate the common tail risk component λ_t by applying the standard Hill (1975) power law estimator to the pooled cross-section of intra-day stock returns[10]:

$$\lambda^{\mathbb{P}}_t = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t}, \tag{2}$$

where $R_{k,t}$ is the kth high-frequency return that is below the threshold u_t on day t, K_t is the total number of returns that fall below this threshold within day t, and the superscript $\mathbb{P}$ denotes that returns are observed under the physical probability measure.[11] The threshold u_t represents an extreme quantile, determining that the observed returns below u_t belong to

[7] See Kelly and Jiang (2014) for a detailed motivation of the use of a power law structure to model the left tail distribution of returns. In sum, for a large class of heavy-tailed distributions, the left tail converges to a generalized power law distribution.

[8] $r < u_t < 0$ and $a_i/\lambda_t > 0$ guarantee that the probability $(r/u_t)^{-a_i/\lambda_t}$ is always between 0 and 1.

[9] In extreme value theory, the parameter λ_t is also often called the shape parameter, and its inverse $1/\lambda_t$ the tail index (see, e.g., Danielsson, 2011).

[10] While Kelly and Jiang (2014) use daily returns to estimate tail risk at a monthly frequency, we rely on intra-day returns to obtain the tail risk specific to day t.

[11] In the Hill formula, returns that fall below threshold u_t are treated as the first K_t entries of R_t. This is without loss of generality since in the pooled cross-section the elements of R_t are exchangeable.

the left tail and follow the power law structure. We define u_t to be the first percentile of the return cross-section for each time period, which makes the threshold time-varying as the pooled intra-day return distribution changes from day to day.

2.2 Risk-neutral Hill estimator

The Hill estimator extracts the common tail risk component from the pooled cross-section of returns observed under the physical probability measure, where all observations are deemed equally likely to happen. In that sense, $\lambda_t^{\mathbb{P}}$ does not incorporate the true risks that are perceived by investors in financial markets. In particular, if investors are risk-averse, then "bad" states of the world where marginal utility is high should be overweighted to reflect compensation for risk. Since such states are precisely the ones that matter for the estimation of tail risk, the economic perception of the left tail of returns may be underestimated by the physical Hill estimator. Moreover, its dynamics will also differ from that captured by $\lambda_t^{\mathbb{P}}$, as compensation for risk demanded by investors varies over time depending on business conditions (see, e.g., Bliss and Panigirtzoglou 2004).

In order to incorporate investors' compensation for risk in the estimation of left-tail risk, we propose a new version of the Hill estimator coupled with risk-neutralization. The idea is to tilt the physical measure such that systematic risk in the cross-section of stock returns is corrected for. This is possible by weighting observations with a pricing kernel, or stochastic discount factor (SDF), that correctly prices systematic factors of high-frequency stock returns. Motivated by Kozak, Nagel, and Santosh (2020), we rely on principal component analysis (PCA) to identify the systematic factors. They show that the absence of near-arbitrage opportunities implies that the SDF can be represented as a function of a few dominant principal components (PCs) of returns. This is also consistent with Pelger (2020), who uses PCA to document the presence of at least five systematic factors explaining the intra-day returns of individual stocks.

More specifically, we consider as systematic factors the top J PCs driving most of the intra-day return variation on day t. Our main analysis sets $J=7$. In following Aït-Sahalia and Xiu (2017) and Pelger (2020), the factor loadings Λ_t are obtained as the eigenvectors associated with the J largest eigenvalues of the realized covariance $R_t^{\top} R_t$, where R_t denotes the panel matrix of the high-frequency log-returns of the stocks. The matrix of intra-day factor returns is then given by $F_t = R_t \Lambda_t$.

Let the vector $F_{n,t}$ denote the return over the n-th intra-daily time interval on day t of the top J PCs of stock returns on day t. We work with an SDF that satisfies the Euler equations for the systematic factors:

$$\frac{1}{N}\sum_{n=1}^{N} m_{n,t} F_{n,t} = 0, \tag{3}$$

where 0 is a conformable vector of zeros and N denotes the total number of intra-daily observations. We normalize the mean of the SDF to be one ($\frac{1}{N}\sum_{n=1}^{N} m_{n,t} = 1$).[12] The pricing kernel tilts the physical measure $1/N$ to produce risk-neutral probabilities $m_{n,t}/N$ that overweight states with high marginal utility to reflect higher compensation for risk demanded by risk-

[12] This implies an implicit gross risk-free rate of one such that we can treat the net stock returns in the cross-section as excess returns.

averse investors. That is, the SDF corrects for risk by risk-neutralizing asset returns with $\tilde{R}^i_{n,t} = m_{n,t} R^i_{n,t}$. We discuss how to identify the pricing kernel in the next subsection.

To derive the risk-neutral Hill estimator $\lambda^{\mathbb{Q}}_t$, we posit that the left tail of the risk-neutral return distribution of each asset i in the cross-section also follows a power law structure. The estimator is then obtained by using the pooled cross-section of risk-neutralized returns in equation (2):

$$\lambda^{\mathbb{Q}}_t = \frac{1}{\tilde{K}_t} \sum_{k=1}^{\tilde{K}_t} \ln \frac{\tilde{R}_{k,t}}{u_t}. \tag{4}$$

Due to the risk-neutralization, negative stock returns observed during states of high (low) marginal utility get properly overweighted (downweighted) by values of the pricing kernel above (below) its mean one, reflecting compensation for risk. The difference between $\lambda^{\mathbb{Q}}_t$ and $\lambda^{\mathbb{P}}_t$ captures the additional tail thickness coming from investors' risk preferences towards extreme negative events.

2.3 Risk-neutralization

The exact distortion of the physical measure, or correction for systematic risk in the cross-section of stocks, depends on the pricing kernel considered.[13] Besides correctly pricing the factor returns, there are two important properties that the SDF must satisfy. First, it must be nonnegative in order to be consistent with no-arbitrage. This guarantees that the tilted risk-neutral probabilities $m_{n,t}/N$ constitute a proper probability measure. Second, it should incorporate information about higher moments of the return distribution. This is important for modeling tail risk, since investors' aversion to downside risk is related to negative skewness aversion (see, e.g., Schneider and Trojani 2015).

We follow the nonparametric approach developed by Almeida and Garcia (2017) to obtain a nonlinear pricing kernel satisfying the properties above. Their method consists in estimating SDFs by minimizing a family of discrepancy loss functions (Cressie and Read, 1984) subject to correctly pricing a set of returns. This approach is a generalization of Hansen and Jagannathan (1991), who show how to obtain a minimum variance SDF from data on asset returns. Almeida and Garcia (2017) consider more general loss functions that take into account higher moments and imply nonnegative SDFs. Adapted to our context, the minimum discrepancy problem is given by:

$$\begin{aligned} &\min_{\{m_{1,t},\dots,m_{N,t}\}} \quad \frac{1}{N}\sum\nolimits_{n=1}^{N} \frac{m_{n,t}^{\gamma+1}-1}{\gamma(\gamma+1)}, \\ &\text{s.t.} \;\; \frac{1}{N}\sum\nolimits_{n=1}^{N} m_{n,t}F_{n,t} = 0, \;\; \frac{1}{N}\sum\nolimits_{n=1}^{N} m_{n,t} = 1, m_{n,t} \geq 0 \forall n, \end{aligned} \tag{5}$$

where $\gamma \in \mathbb{R}$ indexes the convex loss function in the Cressie and Read (1984) discrepancy family. This family captures as particular cases several loss functions in the literature, such as the Hansen and Jagannathan (1991) quadratic loss function when $\gamma = 1$ and the Kullback–Leibler Information Criterion adopted by Stutzer (1995) when $\gamma \to 0$.

[13] We consider the realistic case of an incomplete market, where there exists an infinity of pricing kernels that correctly price the systematic factors under no-arbitrage.

Under the assumption of no-arbitrage in the observed sample, Almeida and Garcia (2017) show that solving equation (5) is equivalent to solving the simpler dual problem below, for $\gamma < 0$[14]:

$$\theta^*_\gamma = \underset{\theta \in \Theta_\gamma}{\operatorname{argmax}} \frac{1}{N} \sum_{n=1}^{N} -\frac{1}{\gamma+1} (1 - \gamma \theta F_{n,t})^{\left(\frac{\gamma+1}{\gamma}\right)}, \tag{6}$$

where $\Theta_\gamma = \{\theta \in \mathbb{R}^J : \text{for } n = 1, \ldots, N, (1 - \gamma\theta F_{n,t}) > 0\}$. The minimum discrepancy SDF can then be recovered from the first-order condition of equation (6) with respect to the row-vector θ, evaluated at θ^*_γ:

$$m^*_{\gamma,n,t} = (1 - \gamma\theta^*_\gamma F_{n,t})^{\frac{1}{\gamma}}, n = 1, \ldots, N. \tag{7}$$

The dual problem can be economically interpreted as an optimal portfolio problem for an investor maximizing hyperbolic absolute risk aversion (HARA) utility, where $\theta^*_\gamma F_{n,t}$ is the endogenous optimal portfolio of the systematic factors. The SDF $m^*_{\gamma,n,t}$ is the marginal utility of the investor and will be higher for "bad" states of nature represented by negative realizations of the optimal portfolio of the factors.

For each γ, the solution θ^*_γ of the dual problem (6) leads to a different minimum discrepancy SDF. While by construction they all correctly price the systematic factor returns, they do so by representing distinct risk preferences. In particular, Almeida and Freire (2022) show that positive absolute prudence (Kimball 1990), which is related to aversion to downside risk and a convex marginal utility, is captured by $\gamma < 1$. Moreover, the smaller the γ, the more aversion to downside risk is embedded in the SDF, where the pricing kernel gets more convex, putting more weight on extreme negative observations of the optimal portfolio returns.[15] They also show that, for extreme negative γs (usually below -5), the constrained maximization in the dual problem (6) may not have a solution. In order to successfully identify a pricing kernel capturing aversion to downside risk, we choose the one associated with $\gamma = -3$ to calculate the risk-neutral Hill estimator.[16]

Overall, our tail measure $\lambda^{\mathbb{Q}}_t$ depends on three hyperparameters: the percentile of the cross-section used to define u_t, the number J of stock PCs treated as systematic factors, and the parameter γ indexing the minimum discrepancy SDF. In Section 4.5, we show that our results are robust across different combinations of a wide range of choices, including percentiles 0.5, 1, and 5 for u_t, $J \in \{1, 3, 5, 7, 10\}$, and $\gamma \in \{-5, -4, -3, -2, -1, 1\}$. Note that when $J = 1$ the implied SDF is akin to a CAPM SDF since the first PC essentially captures the market factor, while $\gamma = 1$ yields a linear SDF (capped at zero to enforce no-arbitrage) with no preferences for higher moments of returns.

Finally, we emphasize that, in line with most of the tail risk literature, we assume investors have rational expectations, meaning their beliefs $\mathbb{B}$ about possible states of the world coincide with the physical measure $\mathbb{P}$. Under this assumption, the SDF obtained in equation (5) reflects investors' risk preferences embedded in the Euler equation. Consequently, $\lambda^{\mathbb{Q}}_t$

[14] For $\gamma > 0$, the problem is unconstrained with an indicator function in the objective function: $\frac{1}{N}\sum_{n=1}^{N} -\frac{1}{\gamma+1}(1-\gamma\theta F_{n,t})^{\left(\frac{\gamma+1}{\gamma}\right)} I_{\Theta_\gamma(F_{n,t})}(\theta)$, where $\Theta_\gamma(F_{n,t}) = \{\theta \in \mathbb{R}^J : (1-\gamma\theta F_{n,t}) > 0\}$ and $I_A(x) = 1$ if $x \in A$, and 0 otherwise. For $\gamma \to 0$, the problem is unconstrained and the objective function is exponential: $\frac{1}{N}\sum_{n=1}^{N} -e^{-\theta F_{n,t}}$.

[15] Since the mean of the pricing kernel continues to be the same, this means that less weight is given to intermediary return observations.

[16] Note that fixing γ to be the same across the sample does not rule out time-varying risk aversion, as the absolute risk aversion of the HARA investor, given by $1/(1-\gamma\theta^*_\gamma F_{n,t})$, will vary across different days as both $F_{n,t}$ and θ^*_γ will change.

incorporates information about both risk and risk prices, that is, about the likelihood of left-tail events and investors' aversion to them. If, instead, the market is populated by a combination of rational and behavioral investors, the SDF pricing the stock PCs could be driven by investor sentiment (Kozak, Nagel, and Santosh 2018), giving $\lambda_t^{\mathbb{Q}}$ the alternative interpretation of capturing subjective beliefs about left-tail risks. Nonetheless, the fact that a simpler CAPM SDF specification, which is commonly used by investors in practice (Graham and Harvey 2001), delivers similar results when used to risk-neutralize returns and construct $\lambda_t^{\mathbb{Q}}$ (see Section 4.5), supports the plausibility of our risk-based interpretation.

3. Data description and implementation details

3.1 Data

Our sample consists of 5-min returns for an unbalanced panel of 875 stocks between January 1996 and December 2022. The data are obtained from Refinitiv Tick History. We sample the 5-min returns using the previous tick interpolation so that each stock has seventy-eight intra-day observations on a given day. In addition, we incorporate the overnight return, yielding a total of seventy-nine observations per day.[17] We apply standard filters and cleaning procedures to the data.[18] In doing so, on any given day, we remove any stocks for which the number of nonzero returns is smaller than thirty-five observations.[19] To illustrate the impact of this filter, figure 1 plots the average number of stocks per year with at least thirty-five nonzero returns per day. As can be seen, this filter has a bigger impact during the years prior to the decimalization of the stocks (April 9, 2001), whereas after this period, the filter mostly eliminates stocks around holidays.

Throughout the article, we use data on market returns, risk factors, and uncertainty measures. The five factors of Fama and French (2015), the momentum factor, and the risk-free rate are obtained from Kenneth French's website. The liquidity factor of Pástor and Stambaugh (2003) is available from Lubos Pastor's website. The *VIX* index is obtained from the Chicago Board Options Exchange (CBOE), while the left-tail volatility (*LTV*) and the right-tail volatility (*RTV*) proposed by Bollerslev, Todorov, and Xu (2015) are computed by ourselves.[20] The *LTV* and *RTV* capture, respectively, the option-implied risk-neutral expectation of return volatility stemming from large negative and positive price jumps. Using high-frequency market returns sampled every 5 min obtained from TickData Inc, we compute measures of the realized variance (*RV*), realized skewness (*RSK*), realized kurtosis (*RK*), and jump variation (*JV*) of the S&P 500 index (Andersen et al. 2001, 2003; Barndorff-Nielsen and Shephard 2004; Amaya et al. 2015). Using daily market returns, we further calculate the reversal (*REV*) of Jegadeesh (1990) and Lehmann (1990), momentum (*MoM*) of Jegadeesh and Titman (1993), and maximum (*Max*) and minimum (*Min*) daily return (Bali, Cakici, and Whitelaw 2011) for the market over the previous week.

We also compute the variance risk premium (*VRP*) as the difference between the risk-neutral and physical expectations of the market return variance (e.g., Bollerslev, Tauchen, and Zhou 2009; Bekaert and Hoerova 2014). We define the *VRP* on day t as the squared *VIX* index (scaled to the monthly level) minus the physical conditional expected value of the future

[17] Results are similar without including the overnight return, as shown in Section 4.6.

[18] To formally exclude the impact of microstructure noise, we have performed the Hausman tests for microstructure noise and first-order serial correlation of Aït-Sahalia and Xiu (2019), for each stock and each day. The tests reject any significant presence of microstructure noise and first-order serial autocorrelation in the returns.

[19] Although this filter mitigates the impact of zero returns (see, e.g., Bandi, Pirino, and Ren 2017, Bandi et al. 2020), our results are robust to setting the threshold for nonzero returns to twenty or fifty observations (see Section 4.6).

[20] We thank Viktor Todorov for making his code available.

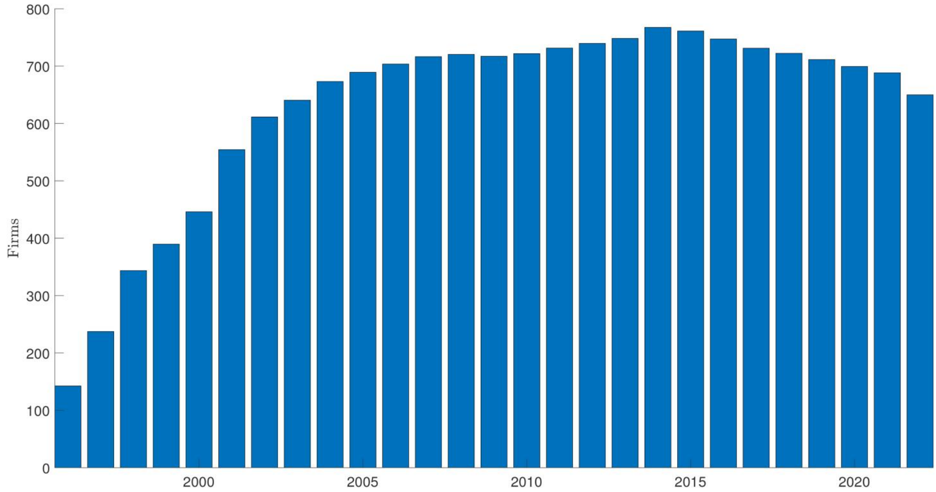

Figure 1 Yearly average number of stocks.

Note: The figure depicts the yearly average number of stocks used to compute the tail risk measures. These stocks have at least thirty-five nonzero returns per day.

monthly realized variance computed using the HAR model of Corsi (2008). Appendix B contains the detailed definitions of the variables we use.

Finally, for our cross-sectional analysis, we obtain the returns and prices for all ordinary common stocks (share codes 10 and 11) from the Center for Research and Security Prices (CRSP) for the period spanning January 1996 to December 2022. In line with the literature, we remove all penny stocks with prices less than five dollars. In the Supplementary Appendix Section OA.5, we show that our results are robust to an even more conservative filter, further removing relatively illiquid stocks with market capitalization smaller than the tenth percentile.

3.2 PCs and risk-neutral estimates

As described in Section 2, for a given day t in our sample, we first extract the top PCs explaining most of the variation in the high-frequency panel of stock returns. The PCs are themselves returns of portfolios of the original stocks. Figure 2 plots the average over each day of our sample of the percentage of variance explained by each of the first ten PCs. The first PC (PC1) explains nearly 30 percent of the variation in the stock returns. In our data, PC1 is always a level factor with positions of similar magnitude across stocks. In other words, as it is usually the case, PC1 can be interpreted as a market factor. The remaining PCs are long–short portfolios of the original stocks.[21] The top-five, seven, and ten PCs together explain around 52, 60, and 65 percent of the return variation across the stocks' returns, respectively.

Then, we estimate the SDF for each day t using the seventy-eight intra-day returns and the overnight return of the top-seven PCs. Given that PC1 can be seen as the market factor, we impose the economic restriction of a 5 percent lower bound on the annualized equity premium, following Almeida and Freire (2022).[22] While results are similar compared to those

[21] Pelger (2020) shows that such higher-order PCs explaining high-frequency stock returns can be interpreted as industry portfolios for specific industries such as oil, finance, and electricity.

[22] For each day t, we impose that the average return of PC1 is at least 5 percent above the risk-free rate, in annualized terms. That is, we shift the mean of PC1 to the lower bound when the bound is binding.

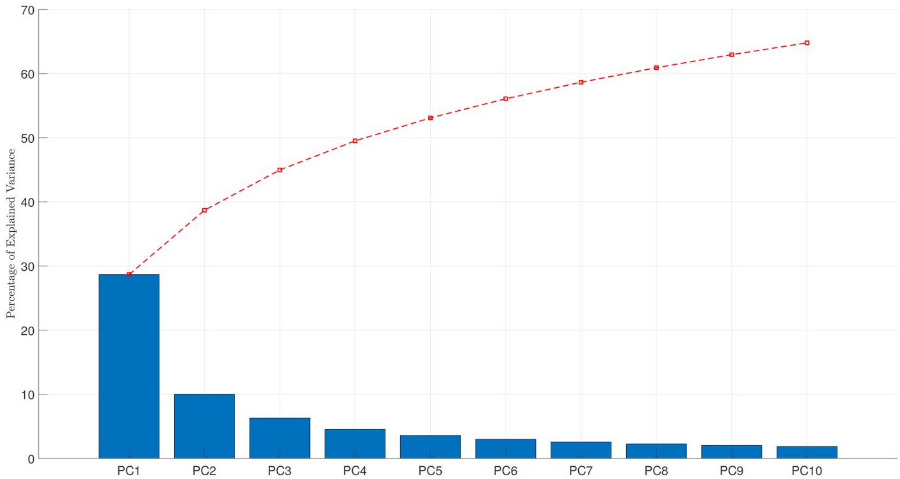

Figure 2 Explained variation of principal components.

Note: The figure depicts the time series average of the percentage of explained variance in the panel of intra-day stock returns by the top-ten PCs (in blue bars) and the accumulated percentage of explained variance (in red). The sample ranges from January 1996 to December 2022.

where this restriction is not imposed, we keep it because it is economically sound to consider a lower bound on the equity premium (Campbell and Thompson 2008; Pettenuzzo, Timmermann, and Valkanov 2014; Martin 2017). This restriction is only imposed for the estimation of the SDF. For the remaining PCs, we do not impose any restrictions.

To illustrate how the SDF distorts the physical measure, figure 3 plots the estimated risk-neutral probabilities ($m_{\gamma,n,t}/N$) for various values of γ and the physical probabilities ($1/N$) for a random day in our sample. The observed patterns are representative of other dates. As can be seen, the risk-neutral measures give more probability weight to negative returns of the optimal portfolio of PCs and less weight to positive returns compared to the physical measure. This reflects agents' risk aversion: investors require more compensation (i.e., the SDF is higher) for "bad" states of the world. In the estimation of tail risk, this is such that negative stock returns, observed during intra-daily intervals for which the optimal portfolio of systematic factors experiences negative (positive) returns, get overweighted (downweighted) to reflect more (less) compensation for risk. The relative compensation for risk in the left tail of the optimal portfolio returns depends, in turn, on the aversion to downside risk. The smaller the γ, the more averse to downside risk (or, equivalently, the more prudent) the investor is, and the greater the weights to negative returns under the risk-neutral measure are. For $\gamma = 1$, prudence is zero and the SDF is linear. We use the SDF associated with $\gamma = -3$ as our baseline specification.

3.3 Tail risk estimates

We estimate the tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ as detailed in Section 2, using the set of intra-day return observations of all stocks available for each day t. The upper panels of figure 4 plot their 1-month moving averages, for ease of exposition. The measures share some similarities. In particular, neither $\lambda_t^{\mathbb{P}}$ nor $\lambda_t^{\mathbb{Q}}$ behave like usual volatility-based risk measures. In fact, the physical tail risk even decreases during the global financial crisis of 2008. To understand this pattern, the left lower panel of figure 4 reports the time-varying threshold u_t (in

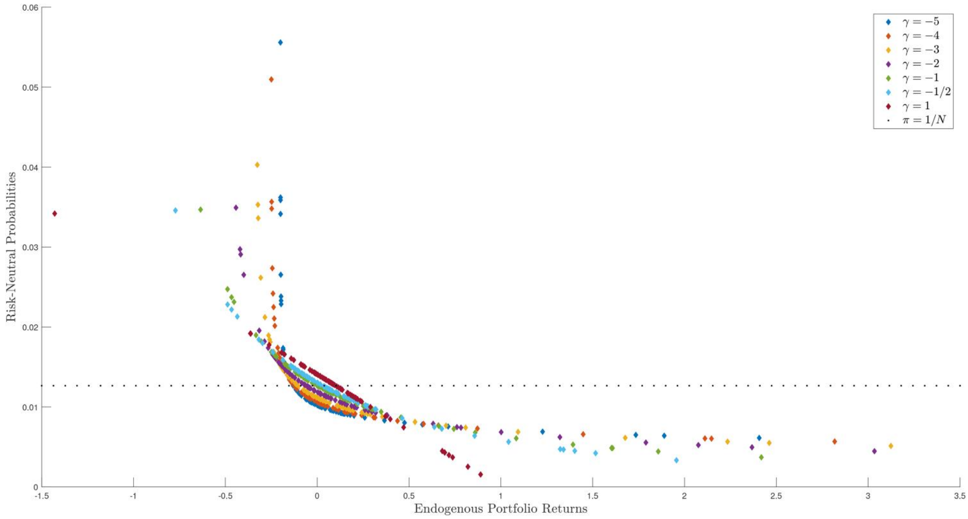

Figure 3 Minimum discrepancy risk-neutral probabilities.

Note: The figure depicts the minimum discrepancy risk-neutral probabilities for various values of γ and the physical measure ($\pi = 1/N$) for the seventy-nine intra-daily endogenous portfolio returns ($\theta_\gamma^* F_{n,t}$) for a random day in our sample.

absolute value), that determines where the left tail begins in the Hill estimator. As can be seen, u_t resembles a volatility measure, peaking during periods of market distress. The tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ can thus be thought of as capturing the thickness of the left tail after taking into account the effect of volatility. As Kelly and Jiang (2014) note, a fixed percentile (of 1 percent of the return cross-section of time t) is used to define u_t exactly for this reason: if volatility increases but the shape of the return left tail is unchanged, an increase of the threshold (in absolute value) absorbs the effect of volatility changes and leaves estimates of the tail exponent unaffected.[23]

Even though the tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ display similarities, they are still fundamentally different. The right lower panel of figure 4 plots the difference between the risk-neutral and physical tail measures. As would be expected, $\lambda_t^{\mathbb{Q}}$ is usually above $\lambda_t^{\mathbb{P}}$, reflecting the fact that investors are averse to downside risk. However, the additional thickness of the left tail coming from risk compensation required by investors varies substantially over time. In particular, $\lambda_t^{\mathbb{Q}} - \lambda_t^{\mathbb{P}}$ tends to peak during crisis periods. This suggests that, even though financial crises are not necessarily associated with higher physical tail risk, they are associated with a higher compensation demanded to bear tail risk. In other words, investors are especially afraid of extreme negative events during periods of market distress.

Table 1 reports summary statistics of and correlations among $\lambda_t^{\mathbb{P}}$, $\lambda_t^{\mathbb{Q}}$, $|u_t|$ and realized and option-implied measures of return variation. The results in the table afford three main conclusions. First, the tail measures have a relatively small correlation of 33.2 percent, reinforcing that they capture substantially distinct information over time. Second, both $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$

[23] In unreported tests, we calculate $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ with a constant threshold $u_t = u$ and find that both measures behave like volatility-type measures. This indicates that defining u_t as a fixed percentile of the return cross-section is instrumental to isolate the effects of volatility from the shape of the left tail. For a similar discussion for longer horizons, see the recent paper by Gormsen and Jensen (2024).

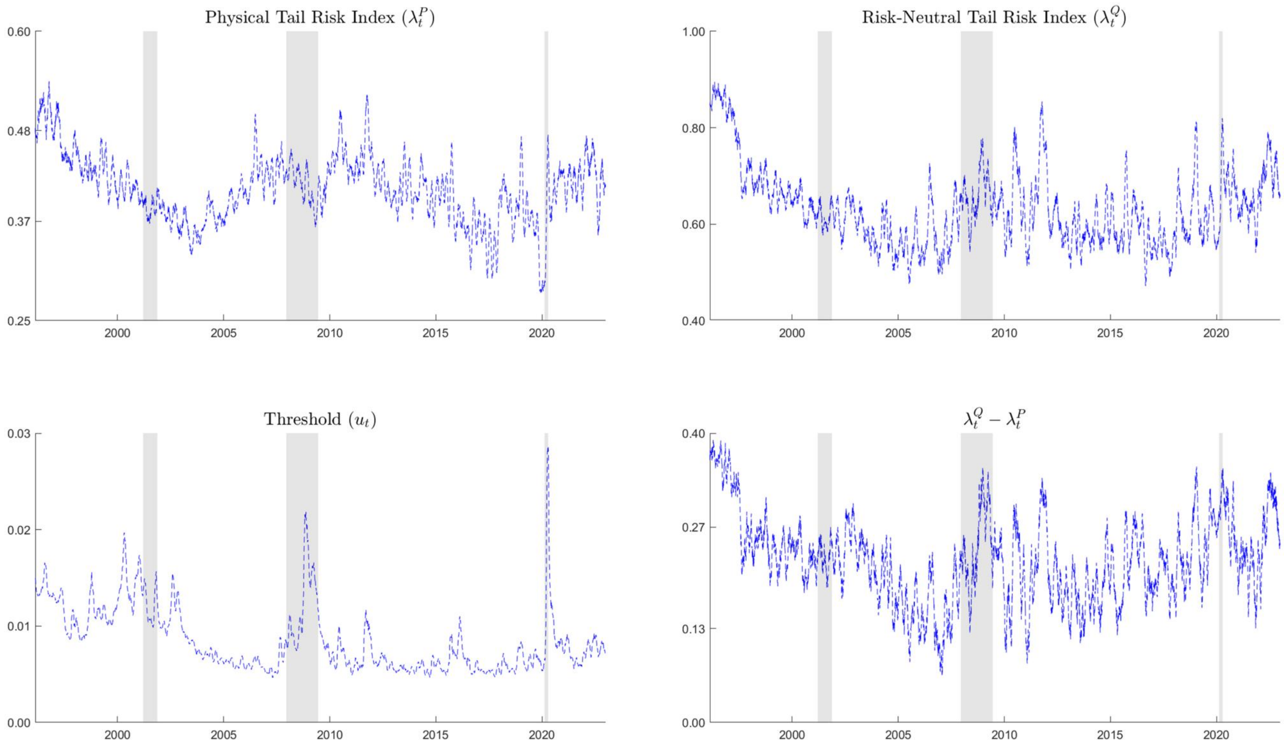

Figure 4 Tail risk index measures.

Note: The figure plots, in the upper panels, the 1-month moving average of the physical and risk-neutral tail risk indices. In the bottom panels, the corresponding moving averages for the (absolute value of the) threshold and the difference between risk-neutral and physical tail risk are depicted. Shaded areas depict the NBER recession dates. The sample ranges from January 1996 to December 2022.

Table 1 Summary statistics.
Note: The table reports, in Panel A, the mean, median, 5th, and 95th percentiles, and the standard deviation of the physical tail risk index ($\lambda_t^{\mathbb{P}}$), the risk-neutral tail risk index ($\lambda_t^{\mathbb{Q}}$), the absolute value of the threshold ($|u_t|$), the realized variance (RV_t) estimated using 5-min intra-day returns, the Jump Variation (JV_t) estimated as the difference between the RV and the bipower variation of Barndorff-Nielsen and Shephard (2004), the left-tail volatility (LTV_t), and the right-tail volatility (RTV_t) of Bollerslev, Todorov, and Xu (2015). Except for $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$, all variables are in percentage terms, and LTV_t and RTV_t are annualized. Panel B reports in the off-diagonal the correlation of each pair of variables and in the main diagonal the AR(1) coefficient. The sample ranges from January 1996 to December 2022.

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	$\lvert u_t \rvert$	RV_t	JV_t	LTV_t	RTV_t
Panel A: Summary statistics							
Mean	0.409	0.617	0.903	1.072	0.074	8.096	0.234
Median	0.408	0.618	0.523	0.544	0.019	7.183	0.134
5th percentile	0.300	0.328	0.218	0.088	0.000	4.038	0.010
95th percentile	0.524	0.897	2.443	3.333	0.288	14.983	0.772
St. Dev.	0.070	0.170	1.723	2.173	0.248	3.714	0.292
Panel B: Correlations and AR(1) coefficients							
$\lambda_t^{\mathbb{P}}$	0.396	0.332	0.082	0.109	0.061	0.104	−0.164
$\lambda_t^{\mathbb{Q}}$		0.308	0.212	0.200	0.123	0.175	−0.141
$\lvert u_t \rvert$			0.756	0.588	0.266	0.347	−0.070
RV_t				0.681	0.525	0.472	−0.064
JV_t					0.113	0.178	−0.031
LTV_t						0.900	−0.055
RTV_t							0.644

are weakly correlated with and less persistent than volatility risk measures, whereas (the absolute value of) u_t has a strong positive correlation with these measures and is highly persistent. This is consistent with the fact that the time-varying threshold u_t controls for the effect of volatility in the calculation of $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$. Third, the risk-neutral tail measure is more volatile than the physical one, which reflects its dependence on variation not only in the level of tail risk, but also in investors' aversion to tail risk.

Finally, the high-frequency nature of our tail measures allows them to react swiftly to market news and capture key economic events. Figure 5 illustrates this by plotting $\lambda_t^{\mathbb{Q}}$ during the global financial crisis, alongside annotations of significant events. We can observe sharp spikes in tail risk during pivotal early events, such as the Bear Stearns bailout and the IndyMac Bank failure. Interestingly, tail risk does not surge as dramatically during the Lehman Brothers collapse, suggesting that after adjusting for volatility, this event did not substantially increase the probability of extreme negative returns. The peak in $\lambda_t^{\mathbb{Q}}$ occurs on November 20, 2008, when the Dow Jones hit a 5-year low. Subsequent interest rate cuts and government stimulus packages are followed by declines in tail risk, which returns to low levels as the crisis stabilizes. The bottom panel depicts the difference between $\lambda_t^{\mathbb{Q}}$ and $\lambda_t^{\mathbb{P}}$ during the same period. The patterns are similar to those of the top panel, indicating that events associated with high tail risk were also linked to heightened tail risk aversion. The main difference is that, while the collapse of Lehman Brothers did not cause a significant spike in the level of tail risk, it did lead to an increase in the compensation required to bear that risk.

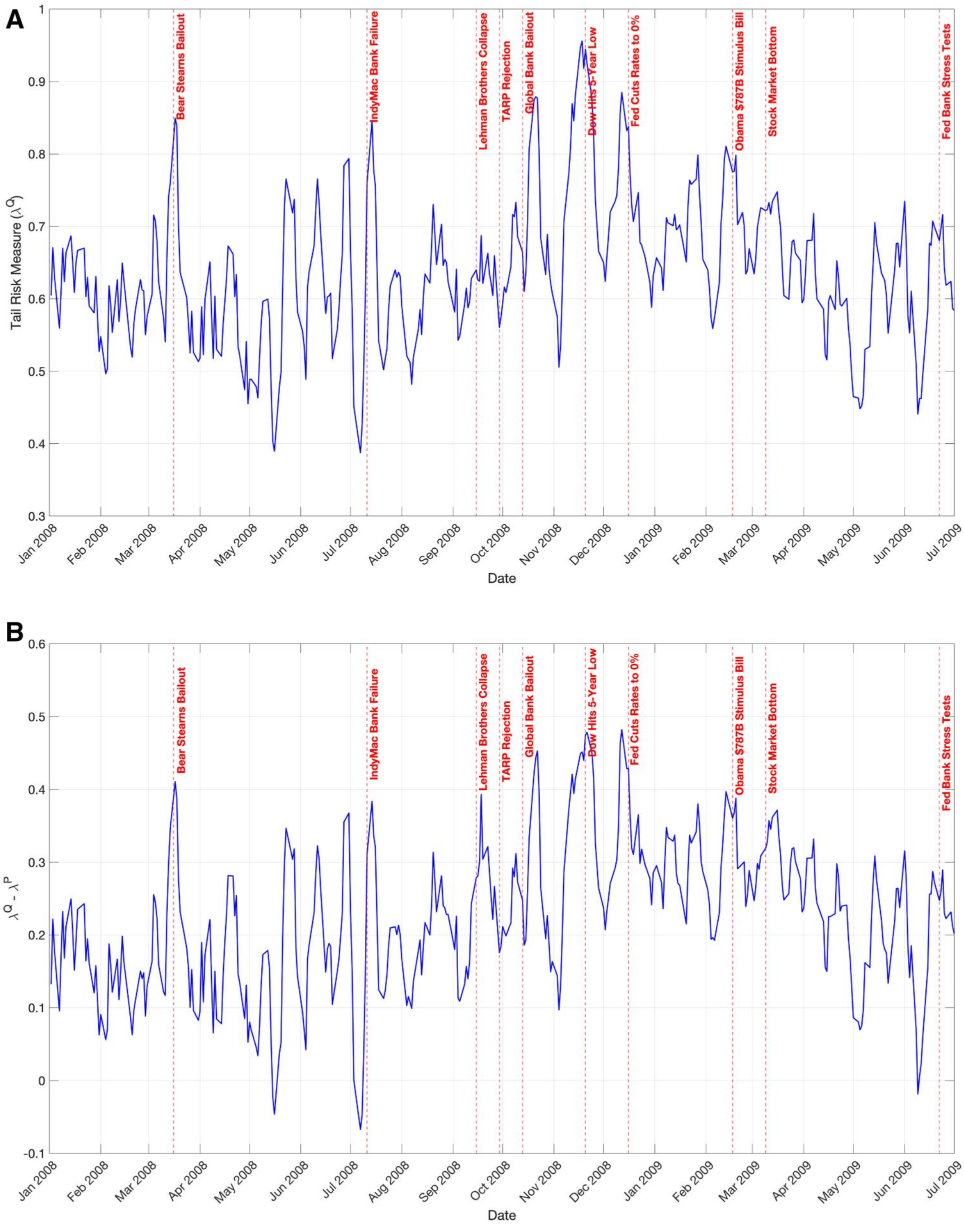

Figure 5 Tail risk measure during the global financial crisis.

Note: The figure plots the 5-day moving average of the risk-neutral tail risk index during the 2008–2009 financial crisis (A) and the difference between the risk-neutral and physical tail risk during the same period (B). Key events are annotated with red dashed lines at their respective dates.

4. Empirical results

This section provides empirical evidence of the information content of our tail risk measures for asset prices in the short-term. We document the predictive power of the tail measures in forecasting the equity premium and the variance risk premium with 1-day ($h = 1$), 1-week ($h = 5$), and 1-month ($h = 22$) ahead daily predictive regressions. In addition, we investigate how tail risk is priced in the cross-section of stocks. To do so, we construct monthly long–short portfolios by sorting stocks on their recent exposure to the tail risk measures, and run Fama and MacBeth (1973) regressions. Then, we provide a comprehensive battery of robustness checks for our main results.

4.1 Equity premium predictability

There is extensive empirical evidence that, at relatively low frequencies (monthly or longer), measures of tail risk strongly predict future excess market returns, with a positive relation (see, e.g., Kelly and Jiang 2014; Bollerslev, Todorov, and Xu 2015; Almeida et al. 2017). This is consistent with the idea that investors are averse to tail risk, such that they require a higher return to hold the market when tail risk increases. To shed light on this relation in the short-term, we investigate whether and how our tail risk measures predict the equity premium at short horizons with daily regressions. We consider regressions of the form $R^e_{t,t+h} = a + bX_t + \varepsilon_{t,t+h}$, where $R^e_{t,t+h}$ is the excess market return accumulated over horizon h and X_t collects the predictors of interest on day t.

Table 2 contains our main results for predicting the equity premium.[24] There is a positive relation between $\lambda^{\mathbb{P}}_t$ and the equity premium. However, this relation is statistically significant at the 5 percent level only at the 1-month horizon. That is, investors require a higher return to hold the market after a tail risk shock, but this compensation appears only over the horizon of 1 month. While $\lambda^{\mathbb{Q}}_t$ is also positively related to future market returns, this relation is strongly significant across all horizons and associated with substantially higher R^2s. This indicates that investors immediately require compensation to hold the market following an increase in their perception of tail risk, where such compensation persists for over a month. To further assess the role of risk-neutralization in predicting the equity premium, we consider predictive regressions including both the risk-neutral and physical tail measures. Across all horizons, $\lambda^{\mathbb{Q}}_t$ remains a significant predictor, while $\lambda^{\mathbb{P}}_t$ has no predictive power. In other words, the information about future market returns contained in the physical tail risk is subsumed by its risk-neutral counterpart.

In Supplementary Appendix Section OA.1, we show that our predictability results for the equity premium are robust to controlling for several alternative predictors. In fact, the risk-neutral tail risk is the strongest predictor of future excess market returns among the controls. The only exception is the *VRP*. Even so, the inclusion of the *VRP* only drives out the effect of $\lambda^{\mathbb{Q}}_t$ at the 1-day horizon. This indicates that the variance risk premium helps explain variation in future excess market returns not only at lower frequencies (Bollerslev, Tauchen, and Zhou 2009), but also in the short-term.

The R^2 values associated with $\lambda^{\mathbb{Q}}_t$ are 0.097, 0.284, and 0.780 percent at the 1-day, 1-week, and 1-month horizons, respectively. To put these magnitudes into perspective, we compute the upper bound on return predictability of Ross (2005) as discussed by Huang and Zhou (2017). Under fairly general conditions, investors' preferences can be bounded by a constant relative risk aversion utility with risk aversion coefficient A. Assuming that the investor with the maximum A has the market portfolio as his wealth that is lognormally distributed, the maximum attainable R^2 in the economy is given by $A^2\sigma^2(R_{mkt})$, where the market return variance depends on the investment horizon. Following Ross (2005) by setting $A = 5$, the implied

[24] The reported coefficients of the predictive regressions are scaled to be interpreted as the effect of a one standard deviation increase in the regressor on future excess market returns.

Table 2 Predicting excess market returns.

Note: The table reports the regression coefficients and robust *t*-statistics of daily predictive regressions for excess returns over 1-day ($h = 1$), 1-week ($h = 5$), and 1-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t+1$ to $t+h$. We compute the *t*-statistics using Newey–West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5%, and 1% level, respectively. The R^2 is the OLS R^2 reported in percentage points. The sample ranges from January 1996 to December 2022.

	1-day ($h = 1$)			**1-week ($h = 5$)**			**1-month ($h = 22$)**		
$\lambda_t^{\mathbb{P}}$	0.029*		0.018	0.064		0.021	0.362**		0.240
t-stat	1.766		1.043	1.550		0.502	2.454		1.580
$\lambda_t^{\mathbb{Q}}$		0.039**	0.033*		0.137***	0.130***		0.446***	0.366***
t-stat		2.413	1.914		3.406	3.227		4.157	3.396
R^2 (%)	0.055	0.097	0.117	0.062	0.284	0.289	0.513	0.780	0.981

upper bounds on predictability are 0.382, 1.65, and 6.25 percent at the 1-day, 1-week, and 1-month horizons, respectively. Relative to these bounds, $\lambda_t^{\mathbb{Q}}$ attains approximately 25.4, 17.2, and 12.5 percent of the maximum possible predictability at the three horizons. These results indicate that the predictive content of $\lambda_t^{\mathbb{Q}}$ is not negligible and is strongest at short horizons, consistent with the high-frequency nature of the information embedded in the measure.

We also investigate whether the predictive power of tail risk holds in a fully out-of-sample exercise. For either $\lambda_t^{\mathbb{Q}}$ or $\lambda_t^{\mathbb{P}}$, we estimate predictive regressions for market returns in a previous window and use these estimates to predict returns accumulated over 1-day, 1-week, 1-month, or 1-quarter horizons ahead in the future. We consider an expanding window scheme with initial window size of 2 years, re-estimating the model each day t. We evaluate predictive power via the $R^2_{oos} = 1 - \sum_t (R_t - \widehat{R}_t)^2 / \sum_t (R_t - B_t)^2$, where $\widehat{R}_t$ is the model's predicted return in period t and B_t, the benchmark, is either the average excess return through time t (Panel A) or set to zero (Panel B).[25] We assess statistical significance based on the Clark and West (2007) statistic, where the null hypothesis is that the regressor has no predictive power. Table 3 reports the out-of-sample results. A positive R^2_{oos} means that the predictor outperforms the benchmark. As can be seen, $\lambda_t^{\mathbb{P}}$ only consistently beats the benchmarks for longer horizons over a month in the future. In contrast, $\lambda_t^{\mathbb{Q}}$ yields positive and statistically significant R^2_{oos}s for all horizons regardless of the benchmark. This reinforces the strong predictive power for the equity premium afforded by the risk-neutral tail risk.

To assess the economic significance of the predictability associated with our tail risk measures, we follow Campbell and Thompson (2008) and consider an investor with mean-variance preferences who optimally chooses the portfolio weight on the market using out-of-sample return forecasts. We impose realistic portfolio constraints that limit short-selling and excessive leverage. Table 4 reports the resulting changes in investor utility relative to the benchmark strategy based on the historical mean forecast of the equity premium. These utility gains are expressed in units of expected annualized return and can therefore be interpreted as the portfolio management fees an investor would be willing to pay each year to exploit the information contained in the predictor. Across a range of risk aversion parameters

[25] The average excess market return is commonly considered as the benchmark in the literature (see, e.g., Campbell and Thompson 2008). However, if the historical mean is a poor predictor of future returns over short horizons, this could inflate our measures of R^2_{oos}. This is why we also include a benchmark of zero, such that the R^2_{oos} simply reflects the relative variation explained by using tail risk as the predictor.

Table 3 Out-of-sample excess market return predictability.

Note: The table reports, in percentage points, the out-of-sample R^2, R^2_{oos}, computed as $R^2_{oos} = 1 - \sum_t (R_t - \widehat{R}_t)^2 / \sum_t (R_t - B)^2$. $\widehat{R}_t$ is the model's predicted return in period t and B, the benchmark, is the average excess market return through t (Panel A) or it is set to zero (Panel B). For forecasting horizons larger than 1 day, we compound the daily return from $t+1$ to $t+h$. We also denote with *, **, and *** significance at the 10%, 5%, and 1% level, respectively, based on the Clark and West (2007) statistic, whose null hypothesis is that the additional regressor has no predictive power. The models are estimated on expanding windows, with an initial window size of 528 days (equivalent to approximately 24 months).

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$
Panel A: Benchmark $\widehat{\mu}_t$		
One-day ($h=1$)	0.022	0.025**
One-week ($h=5$)	0.004	0.205***
One-month ($h=22$)	0.436**	0.729***
One-quarter ($h=66$)	1.196**	1.565**
Panel B: Benchmark $\widehat{\mu}=0$		
One-day ($h=1$)	0.014*	0.018**
One-week ($h=5$)	0.133**	0.333***
One-month ($h=22$)	1.001***	1.292***
One-quarter ($h=66$)	2.892***	3.254***

and portfolio constraints, trading strategies based on $\lambda_t^{\mathbb{Q}}$ generate utility gains that are generally positive and consistently larger than those obtained using $\lambda_t^{\mathbb{P}}$. For example, at the 1-day horizon, investors are willing to pay annual fees between 1.6 and 3 percent to exploit the predictability afforded by the risk-neutral tail risk measure. These results reinforce that the predictive content of $\lambda_t^{\mathbb{Q}}$ is not only statistically significant but also economically meaningful.

In sum, our results indicate that the economic perception of tail risk is an important determinant of the equity premium in the short-term. Investors require a significantly higher market return following an increase in their perception of tail risk. This effect persists over time, is robust to controlling for several alternative predictors, holds out-of-sample, and is economically significant. Importantly, accounting for investors' aversion to downside risk in computing tail risk provides fundamental information about the equity premium that is not contained in the physical tail risk measure.

4.2 Variance risk premium predictability

The variation of volatility is often associated with time-varying economic uncertainty. In particular, the *VRP* captures investors' compensation for variance risk and is usually regarded as a proxy for aggregate risk aversion (see, e.g., Campbell and Cochrane 1999; Bekaert, Hoerova, and Duca 2013). Bollerslev, Todorov, and Xu (2015) show that a large fraction of the variance risk premium comes from compensation demanded by investors for bearing left-tail risk. Motivated by that, we examine the predictive relation between our tail risk measures and the *VRP* at short horizons, with particular focus on the role of incorporating investors' aversion to downside risk with risk-neutralization.

Table 5 reports the main predictability results for the variance risk premium. Both $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ significantly and positively predict the *VRP*, consistent with the idea that investors require a higher compensation to bear variance risk when tail risk increases. The predictive power of the risk-neutral tail measure is stronger, as indicated by larger t-statistics and individual R^2s.

Table 4 Economic value of tail risk measures.

Note: The table reports the annualized certainty equivalent return (CER) gains (in percentage points) from using tail risk measures to predict excess market returns relative to a historical mean benchmark. A mean-variance investor allocates between the market return and the risk-free rate with portfolio weights $\frac{w_t = \widehat{R}_{t,t+h}}{\gamma \widehat{\sigma}_{t,t+h}2}$, where $\widehat{R}_{t,t+h}$ is the predicted return and $\widehat{\sigma}_{t,t+h}2$ is the EWMA variance forecast. The investor's utility is $U_t = R_{p,t} - A/2(R_{p,t} - \bar{R_p})^2$, where A is the level of risk aversion, and CER gains are computed as the average utility differential between each predictive model and the benchmark. The portfolio is rebalanced every h days using nonoverlapping returns. Columns correspond to forecast horizons ranging from one day to one quarter. Panel A imposes long-only constraints with weights bounded between 0 and 1; Panel B allows short positions and leverage with weights in $[-0.15, 1.5]$; Panel C permits long-only positions with leverage up to 175%.

	λ_t^P	λ_t^Q	λ_t^P	λ_t^Q	λ_t^P	λ_t^Q	λ_t^P	λ_t^Q
	$h = 1$		$h = 5$		$h = 22$		$h = 66$	
Panel A: $w_t \in [0, 1]$								
$A = 2$	0.892	1.619	−0.035	0.528	−0.309	1.262	0.629	1.563
$A = 6$	1.375	1.628	−0.372	0.488	−0.960	−0.177	0.180	0.624
$A = 10$	1.177	1.761	−0.189	0.582	−0.278	−0.049	−0.736	−0.238
Panel B: $w_t \in [-0.15, 1.5]$								
$A = 2$	1.821	1.981	−0.674	−0.019	−1.093	1.509	1.402	2.608
$A = 6$	1.934	2.657	−0.322	0.811	−0.679	−0.318	−1.133	−0.127
$A = 10$	1.007	1.991	−0.313	0.432	0.091	0.192	−1.192	−0.493
Panel C: $w_t \in [0, 1.75]$								
$A = 2$	1.882	1.970	−0.861	−0.162	−0.980	1.424	1.676	2.882
$A = 6$	1.989	3.001	−0.334	1.006	−0.437	−0.036	−1.350	−0.641
$A = 10$	0.918	2.341	−0.135	0.514	0.261	0.271	−1.082	−0.480

Importantly, when both tail measures are included in the regressions, only $\lambda_t^{\mathbb{Q}}$ is significant. This indicates that, again, the investors' perception of tail risk plays a prominent role in explaining aggregate market risk premia in the short-term. In Supplementary Appendix Section OA.1, we show that these results are robust to controlling for a number of alternative predictors, including realized and option-implied measures of continuous and jump variation. Together with $\lambda_t^{\mathbb{Q}}$, the strongest predictor is the *LTV* from Bollerslev, Todorov, and Xu (2015), which captures the expected volatility stemming from negative price jumps implied by short-dated options. In other words, the perception of tail risk extracted from the cross-section of high-frequency returns offers complementary information to *LTV* about the future *VRP* over short horizons.

In sum, we document that our risk-neutral tail measure contains strong predictive power for the variance risk premium in the short-term. Investors require a higher compensation to bear variance risk when their perception of tail risk increases. These effects are robust to several measures of volatility, jump risk, and the *LTV* of Bollerslev, Todorov, and Xu (2015). In contrast, the relation between physical tail risk and future *VRP* is weaker and often statistically insignificant, depending on the controls.

4.3 Cross-sectional return predictability

So far, we have shown that the economic perception of tail risk by investors is an important determinant of aggregate market risk premia at short horizons. We now investigate whether

Table 5 Predicting variance risk premium (VRP).

Note: The table reports regression coefficients and robust t-statistics of daily predictive regressions for the variance risk premium (VRP) over 1-day ($h=1$), 1-week ($h=5$), and 1-month ($h=22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t+1$ to $t+h$. We compute the t-statistics using Newey–West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5%, and 1% level, respectively. The R^2 is the OLS R^2 reported in percentage points. The sample ranges from January 1996 to January 2022.

	1-day ($h=1$)			**1-week ($h=5$)**			**1-month ($h=22$)**		
$\lambda_t^{\mathbb{P}}$	1.831***		0.441	9.527***		2.662	35.450***		6.007
t-stat	6.137		1.499	4.970		1.468	3.033		0.541
$\lambda_t^{\mathbb{Q}}$		4.210***	4.059***		20.954***	20.043***		88.023***	85.965***
t-stat		13.002	12.495		10.012	9.831		6.586	6.598
R^2 (%)	0.867	4.579	4.624	1.078	5.215	5.289	0.874	5.390	5.412

recent exposure to tail risk is priced in the cross-section of stock returns through portfolio sorts. To do so, at the end of each month in our CRSP sample, we measure the insurance value of the stocks with daily regressions over a short previous window of 24 months, that is, we estimate contemporaneous betas with respect to our tail measures: $R_{i,t} = \mu_i + \beta_i TR_{i,t} + \varepsilon_{i,t}$, where $TR_{i,t} \in \{\lambda_t^{\mathbb{P}}, \lambda_t^{\mathbb{Q}}\}$.[26] Then, we form equally-weighted portfolios by sorting the stocks into portfolios using quintile breakpoints calculated based on the given sorting variable and compute the portfolios' returns over the next month.[27] This yields a monthly time series of returns for each portfolio over our sample.

Panels A and B of Table 6 report the results for $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$, respectively. In addition to the average returns of the quintile portfolios, we also report the portfolios alphas (i.e., intercepts) from monthly regressions of portfolio excess returns on the Fama–French three and five factors as well as extended models controlling for momentum (Carhart 1997) and liquidity (Pástor and Stambaugh 2003) factors, and a downside risk factor computed analogously to our tail factors but sorting on betas with respect to *LTV*. The last two columns report the average returns and alphas of the high minus low zero net investment portfolio and associated t-statistics, which are estimated using Newey–West robust standard errors. Panel C presents the P-values from tests of the monotonicity (Patton and Timmermann 2010) of average returns across the five quintile portfolios reported in Panels A and B. All tests have a null hypothesis of a flat pattern (no relation). While the MR Up and MR Down tests have alternative hypotheses of an increasing and decreasing pattern, the MR test alternative hypothesis is unrestricted. These tests are estimated using 10,000 bootstrap replications and a block length equal to 10 months.

Several conclusions can be drawn from these results. First, stocks that are more positively related to tail risk in the short-term earn lower returns. This is economically sound, as stocks with high β_i provide hedging opportunities against tail risk and are thus highly priced, yielding subsequent low returns. This relation is monotonic across quintile portfolios for $\lambda_t^{\mathbb{Q}}$, which is formally confirmed by the rejection of the flat pattern using the MR and MR Down tests. In contrast, a flat pattern cannot be rejected for $\lambda_t^{\mathbb{P}}$. In fact, in this case, it is clear that the average return decreases from the low quintile to the fourth one, but then increases for the high quintile.

[26] Our results are robust to different estimation windows for the betas, such as 18 or 36 months, as shown in Tables OA.36 and OA.37 in the Supplementary Appendix Section OA.5.
[27] Our results are robust to forming value-weighted portfolios instead, as shown in Section 4.6.

Table 6 Monthly sorted portfolios.

Note: The table reports the results of univariate portfolio analyses of the relation between the tail risk measures and the cross-section of returns. Monthly portfolios are formed by sorting the CRSP stocks with share code 10 and 11, excluding those with a share price below \$5, into portfolios using quintile breakpoints calculated based on the given sort variable. The table also reports portfolio alphas from regression of portfolio excess return using the Fama–French three and five factors as well as extended models controlling for momentum (Carhart 1997) and liquidity (Pástor and Stambaugh 2003) factors, and a downside risk factor based on sorting stocks with respect to their LTV_t betas. Returns and alphas are in percentage terms. The last two columns report the high minus low zero net investment portfolio and associated *t*-statistics, which are estimated using Newey–West robust standard errors with a lag length equal to 5. Panel C presents the *P*-values from various tests of the monotonicity (Patton and Timmermann 2010) of average returns across the five quintile portfolios reported in Panels A and B. All tests have a null hypothesis of a flat pattern (no relation). While the MR Up and MR Down tests have alternative hypotheses of an increasing and decreasing pattern, the MR test is unrestricted. Bold *P*-values indicate significance at the 5% or better. The tests are estimated using 10,000 bootstrap replications and a block length equal to 10 months. The sample ranges from January 1998 to December 2022.

	Low	2	3	4	High	High-Low	*t*-stat
Panel A: $\lambda_t^{\mathbb{P}}$							
Average Return	0.786	0.577	0.567	0.498	0.648	−0.138	−0.880
CAPM Alpha	0.056	−0.026	0.013	−0.069	0.015	−0.041	−0.264
FF3 alpha	−0.013	−0.105	−0.068	−0.157	−0.076	−0.063	−0.463
FF5 alpha	0.118	−0.154	−0.157	−0.234	−0.052	−0.170	−1.275
FF5 + Mom alpha	0.144	−0.124	−0.137	−0.200	−0.001	−0.145	−1.054
FF5 + Mom + Liq alpha	0.116	−0.144	−0.154	−0.213	0.001	−0.115	−0.796
FF5 + Mom + Liq + LTV alpha	0.118	−0.144	−0.155	−0.213	0.000	−0.118	−0.860
Panel B: $\lambda_t^{\mathbb{Q}}$							
Average Return	1.173	0.657	0.520	0.419	0.308	−0.865	−5.004
CAPM Alpha	0.506	0.078	−0.041	−0.167	−0.386	−0.892	−4.741
FF3 alpha	0.413	−0.010	−0.123	−0.249	−0.448	−0.861	−4.987
FF5 alpha	0.410	−0.107	−0.213	−0.302	−0.266	−0.676	−4.503
FF5 + Mom alpha	0.436	−0.083	−0.194	−0.267	−0.210	−0.647	−3.571
FF5 + Mom + Liq alpha	0.405	−0.111	−0.214	−0.288	−0.185	−0.590	−2.997
FF5 + Mom + Liq + LTV alpha	0.408	−0.110	−0.215	−0.289	−0.188	−0.596	−4.192
Panel C: Monotonicity Test							
	MR	MR Up	MR Down				
Avg. return $\lambda_t^{\mathbb{P}}$	0.830	0.254	0.131				
Avg. return $\lambda_t^{\mathbb{Q}}$	**0.006**	0.928	**0.000**				

Second, exposure to physical tail risk generates insignificant average returns for the high minus low portfolio. On the other hand, the return spreads associated with exposures to risk-neutral tail risk are both statistically and economically significant, where the corresponding high minus low strategies earn an average monthly return of −0.86 percent. This shows that recent exposure to tail risk as perceived by investors is priced in the cross-section, that is, investors are willing to pay a premium to be hedged against tail risk. To further illustrate, figure 6 plots the cumulative returns of the quintile portfolios based on $\lambda_t^{\mathbb{Q}}$. As can be seen, the robust profitability of selling the high minus low strategy is mainly driven by going long in the stocks that have high expected returns because they do not help hedge against tail risk.[28] The strategy is particularly profitable during crisis periods, reflecting the hedging effectiveness of the tail factor. During such times, stocks that provide protection against tail risk become especially more expensive relative to those with low tail risk betas.

Third, the average high minus low returns of the risk-neutral tail factor in Table 6 survive after controlling for standard factor models in the literature. The alphas with respect to the market and the Fama–French size and value factors are largely of the same magnitude and significance of the original average returns. The additional profitability, investment, momentum and liquidity factors hold some explanatory power to the short-term tail risk premium, reducing (in absolute value) the average return to −0.590 percent. Even so, this still roughly amounts to a 7 percent annualized (absolute) alpha, which is strongly significant with a *t*-statistic of 3. This suggests that our risk-neutral tail factor captures compensation for risk in the cross-section of returns that is not reflected in firms' exposures to these popular factors in the literature. Further adding a downside risk factor based on *LTV* cannot explain the tail factor risk premium. In fact, it actually increases the alpha and its statistical significance (in absolute terms).

As an additional test, we also run Fama and MacBeth (1973) regressions. For each month, for each of the tail measures and controls we consider, we first estimate, for each stock in our CRSP sample, contemporaneous betas with respect to the variable of interest with a daily univariate regression over a short previous window of 24 months (as we are interested in the effect of short-term exposure to the factors). Then, for each month, we run a cross-sectional multivariate regression of next-month stock excess returns on the betas of the variables of interest. Finally, after repeating this procedure for all months in the sample, we test whether the average of the coefficient across months of a given variable is statistically significant with Newey–West standard errors.

Table 7 contains the results. The first column focuses on the Fama–French five-factor model, where only recent exposure to the market and the size factor are significant, although only at a 10 percent level. Columns 2 and 3 add the physical and risk-neutral tail risk, respectively. Both are strongly significant, with a negative sign: stocks with higher tail risk betas, and thus higher hedging capacity, are highly priced and hence have low expected returns. When we include both tail measures together in column 4, the effect of $\lambda_t^{\mathbb{Q}}$ remains essentially the same, while that of $\lambda_t^{\mathbb{P}}$ is largely attenuated. For the remaining columns, we add different controls, one by one, and all of them in the last column. The effect of the economic perception of tail risk is largely unaffected by the inclusion of the controls, remaining strongly significant across all specifications. The physical tail risk, on the other hand, is driven out by LTV_t and RTV_t.

Moreover, given the importance of the *VRP* in the prediction of the equity premium, we use conditional double sorts to specifically investigate whether controlling for it can account for the cross-sectional effects of $\lambda_t^{\mathbb{Q}}$. We first sort stocks into quintile portfolios based on their exposure to *VRP*, and then, within each quintile, we sort stocks into quintile portfolios based on their exposure to tail risk. Table 8 reports the results. Focusing first on Panel A, sorting based on $\lambda_t^{\mathbb{P}}$ only generates significant spreads in average returns for stocks in the highest

[28] It is worth noting that, although the cumulative return is higher for the low portfolio compared to the low minus high strategy, the former is a long-only portfolio and therefore more exposed to other risk factors. In fact, as shown in Table 6, while the low portfolio delivers a higher average return, the low minus high strategy exhibits significantly larger alphas. Furthermore, figure 6 demonstrates that the low minus high strategy achieves comparable returns with lower volatility, resulting in a substantially higher Sharpe ratio: 1.12 annualized versus 0.58 for the low portfolio.

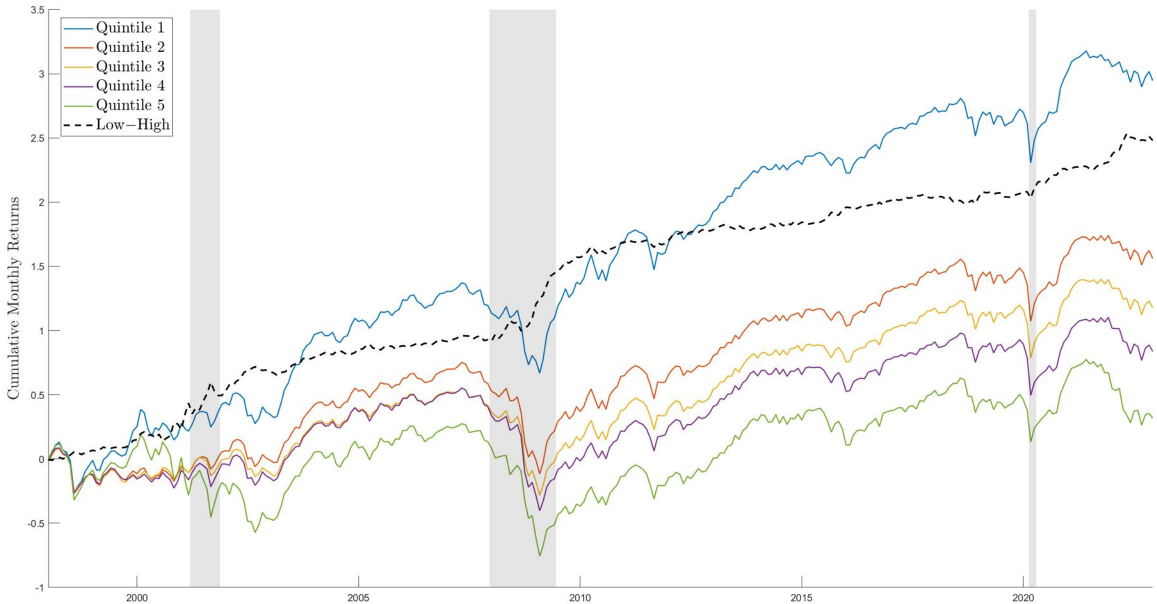

Figure 6 Cumulative monthly quintile portfolio returns formed by sorting on $\lambda_t^{\mathbb{Q}}$.

Note: The figure depicts the cumulative monthly returns for each quintile portfolio formed by sorting on the risk-neutral tail risk measure ($\lambda_t^{\mathbb{Q}}$) and for the low minus high strategy. Shaded areas depict NBER recession dates. The sample ranges from January 1998 to December 2022.

Table 7 Fama–MacBeth Regressions.

Note: The table reports coefficients and robust t-statistics (in parentheses) from Fama and MacBeth (1973) regressions of stock excess returns on betas with respect to different variables, which differ across columns. For each month and for each variable, we first estimate, for each stock in our CRSP sample, contemporaneous betas with respect to the variable with a daily univariate regression over a short previous window of 24 months. Then, for each month, we run a cross-sectional multivariate regression of next-month stock excess returns on the betas of the variables of interest, without intercept. For ease of interpretation, betas are cross-sectionally standardized to have unit variance. Finally, after repeating this procedure for all months in the sample, we test whether the average of the coefficient across months of a given variable is statistically significant with Newey–West robust standard errors with a lag length equal to 5. The sample ranges from January 1998 to December 2022.

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
MKT	0.002*	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.000	0.001	0.001*	0.000
	(1.813)	(1.473)	(1.512)	(1.437)	(1.104)	(−0.128)	(1.037)	(1.386)	(1.423)	(−0.764)	(1.250)	(1.730)	(−0.605)
SMB	0.002*	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.000
	(1.703)	(1.417)	(1.525)	(1.339)	(1.197)	(1.168)	(1.168)	(1.349)	(1.256)	(0.243)	(1.195)	(1.533)	(0.135)
HML	0.000	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	(0.397)	(1.129)	(1.036)	(1.372)	(1.541)	(0.407)	(1.193)	(1.401)	(1.347)	(0.904)	(1.264)	(1.370)	(0.239)
CMA	0.000	0.000	0.000	0.000	0.000	−0.001	0.000	0.000	0.000	−0.001	0.000	0.000	−0.001
	(0.426)	(−0.052)	(0.174)	(−0.281)	(−0.491)	(−0.983)	(−0.312)	(−0.390)	(−0.110)	(−1.784)	(−0.422)	(−0.171)	(−1.562)
RMW	0.000	0.001	0.000	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.396)	(1.062)	(0.587)	(0.717)	(0.897)	(1.571)	(0.469)	(0.699)	(0.527)	(−0.003)	(0.283)	(0.613)	(0.046)
$\lambda_t^{\mathbb{P}}$		−0.003***		−0.001**	−0.001	−0.001	−0.001*	−0.001**	−0.001**	−0.001**	−0.001**	−0.001**	0.000
		(−2.813)		(−1.965)	(−1.087)	(−1.164)	(−1.906)	(−2.017)	(−2.043)	(−1.999)	(−2.229)	(−2.067)	(−1.291)
$\lambda^{\mathbb{Q}}$			−0.004***	−0.003***	−0.002***	−0.002***	−0.003***	−0.003***	−0.003***	−0.001***	−0.003***	−0.003***	−0.001***
			(−3.895)	(−3.480)	(−3.032)	(−2.931)	(−3.348)	(−3.518)	(−3.665)	(−3.170)	(−3.663)	(−3.416)	(−3.275)
LTV					−0.002**								0.000
					(−2.473)								(−0.372)
RTV						−0.004***							−0.003***
						(−3.576)							(−3.400)

(continued)

Table 7 (continued)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
RV							0.002**						0.001
							(2.227)						(1.581)
RSK								0.000					0.000
								(−0.270)					(−0.474)
RKU									0.000				0.000
									(0.724)				(−1.409)
MIN										0.005***			0.003**
										(3.447)			(2.349)
MAX											0.000		−0.001
											(0.405)		(−1.276)
JV												−0.001	0.001*
												(−1.254)	(1.832)

Table 8 Double-sorted portfolios controlling for *VRP*.

Note: The table reports the results for double-sorted portfolio analyses of the relation between tail risk measures and the cross-section of returns after controlling for *VRP*. Monthly portfolios are formed by first sorting the CRSP stocks with share code 10 and 11, excluding those with a share price below \$5, into portfolios using quintile breakpoints calculated based on their exposure to *VRP*. Then, within each quintile, we sort stocks into quintile portfolios based on their exposure to tail risk. The table also reports the alpha from the regression of portfolio excess returns using the Fama–French 5 factors augmented with momentum (Carhart 1997), liquidity Pástor and Stambaugh (2003), and a downside risk factor based on sorting stocks with respect to their LTV_t betas, that is, FF5 + Mom + Liq +LTV α. Returns and alphas are in percentage terms. t-statistics are estimated using Newey–West robust standard errors with a lag length equal to 5. The sample ranges from January 1998 to January 2022.

		Low	2	3	4	High	H-L	*t*-stat	α	*t*-stat
Panel A: $\lambda_t^{\mathbb{P}}$										
VRP	Low	0.842	0.575	0.547	0.640	0.737	−0.105	−0.483	0.062	0.282
	2	0.838	0.701	0.582	0.620	0.713	−0.125	−0.618	−0.006	−0.029
	3	0.906	0.629	0.587	0.522	0.701	−0.204	−1.231	−0.101	−0.591
	4	0.917	0.622	0.636	0.627	0.654	−0.263	−2.031	−0.234	−1.758
	High	0.964	0.713	0.517	0.563	0.787	−0.176	−1.219	−0.284	−2.067
Panel B: $\lambda_t^{\mathbb{Q}}$										
VRP	Low	1.408	0.829	0.454	0.312	0.346	−1.061	−3.764	−0.833	−3.322
	2	1.074	0.685	0.703	0.509	0.487	−0.586	−3.938	−0.397	−2.340
	3	0.979	0.660	0.608	0.579	0.520	−0.458	−3.131	−0.356	−2.682
	4	1.046	0.656	0.568	0.585	0.604	−0.442	−3.880	−0.425	−3.871
	High	1.144	0.705	0.508	0.518	0.672	−0.472	−4.356	−0.531	−4.785

quintiles of *VRP*. In contrast, Panel B reveals that sorting on $\lambda_t^{\mathbb{Q}}$ yields highly significant average returns and alphas regardless of the *VRP* quintile, with the strongest effects concentrated in the low- and high-*VRP* stocks. Therefore, the strong cross-sectional predictability associated with the economic valuation of tail risk is not subsumed by the variance risk premium.

Finally, to learn about which types of firms load more on tail risk and contribute to hedge against it, we report in Table 9 the average values of standard firm characteristics across quintiles formed by sorting stocks on our tail measures. While no clear patterns appear for $\lambda_t^{\mathbb{P}}$, stocks with high exposure to $\lambda_t^{\mathbb{Q}}$ tend to be more liquid (lower Amihud illiquidity) and less levered. These findings are intuitive. Low-leverage firms are less exposed to distress and refinancing risk, while liquid firms are less susceptible to liquidity spirals and fire-sale dynamics during market stress. These stocks are therefore more resilient in stressed market conditions and provide relatively better payoffs in left-tail states, making them natural hedging assets. Other characteristics such as size, book-to-market, market beta, idiosyncratic volatility, realized minus implied volatility, bid–ask spread, profitability, or asset growth show no systematic relation across quintiles.

In sum, we find that the investors' perception of tail risk in the short-term is strongly priced in the cross-section of stocks. High minus low portfolios based on the recent exposure to tail risk generate statistically and economically significant average returns, even after controlling for standard factor models. Such recent exposure to tail risk is also strongly significant in Fama–MacBeth regressions, including a large set of controls. The information content of risk-neutralization beyond that contained in physical tail risk is especially relevant for these findings.

Table 9 Portfolio characteristics.
Note: The table reports for each quintile the average across the months in the sample of the median values within each month of various characteristics for the stocks. We consider market capitalization (in millions of dollars), the log book-to-market (BM) ratio, the market beta, the Amihud Illiquidity (scaled by 10^6), the idiosyncratic volatility from Fama–French 3 factors, realized volatility minus implied volatility (*RV – IV*), Bid–Ask spread, operating profitability, asset growth, and leverage. Characteristics are obtained from the Open Source Asset Pricing Website. The sample ranges from January 1998 to December 2022.

	Low	2	3	4	High
Panel A: $\lambda_t^{\mathbb{P}}$					
Size ($\$10^6$)	491.350	1,245.693	1,366.739	1,149.752	640.787
Log BM	−1.002	−0.997	−0.995	−0.995	−1.006
Beta	1.067	0.824	0.754	0.754	0.915
Illiquidity (10^6)	0.021	0.011	0.010	0.010	0.015
Idio. Vol	0.022	0.016	0.015	0.015	0.019
RV–IV ($\times 100$)	−3.021	−2.986	−2.977	−2.992	−3.031
Bid–Ask Spread	0.011	0.008	0.008	0.008	0.010
Oper. Profitability	0.213	0.228	0.230	0.234	0.216
Asset Growth	0.091	0.079	0.076	0.078	0.088
Leverage	0.532	0.652	0.720	0.684	0.527
Panel B: $\lambda_t^{\mathbb{Q}}$					
Size ($\$10^6$)	428.836	1,203.451	1,441.749	1,217.799	668.873
Log BM	−0.965	−0.980	−0.996	−1.007	−1.034
Beta	0.990	0.807	0.762	0.778	0.984
Illiquidity (10^6)	0.044	0.017	0.009	0.007	0.010
Idio. Vol	0.021	0.016	0.015	0.016	0.020
RV–IV ($\times 100$)	−3.023	−2.991	−2.961	−2.954	−3.021
Bid–Ask Spread	0.010	0.008	0.008	0.008	0.010
Oper. Profitability	0.215	0.228	0.232	0.227	0.210
Asset Growth	0.085	0.079	0.076	0.078	0.096
Leverage	0.652	0.694	0.690	0.639	0.471

4.4 Is the risk-neutral tail measure subsumed by the SDF?

The construction of our risk-neutral tail risk measure relies on an SDF to risk-neutralize intra-day stock returns. This naturally raises the question of why it is necessary to construct a separate measure that embeds information from the SDF to predict returns, rather than using the SDF itself, which in principle prices assets. The key reason is that the SDF in equation (7) is estimated conditionally on the intra-day realizations of stock PC returns within a given day and is designed to price those returns. These SDF realizations are inherently intra-day objects: they pertain only to the states observed within day t and therefore cannot be directly used to price or predict future returns at daily, weekly, or monthly horizons. Rather, we use $m_{n,t}$ to identify intra-day states that are associated with high or low marginal utility, and then use this information for constructing $\lambda_t^{\mathbb{Q}}$. In other words, we aggregate information from risk-neutralized intra-day returns into a single daily measure that captures the thickness of the left tail while explicitly accounting for investors' preferences. We then use $\lambda_t^{\mathbb{Q}}$ as a daily state variable to explain future market and cross-sectional returns over multiple horizons.

Our focus on $\lambda_t^{\mathbb{Q}}$ as the variable of interest reflects our goal of assessing the short-term effects of tail risk on asset prices. Alternatively, one could consider constructing a different daily statistic by aggregating information from the intra-day SDF directly. However, there is no guarantee that such an object would successfully price or predict returns at longer horizons, since the SDF is only required to price the intra-day PCs returns on day t. It is an empirical question whether such a measure would be informative about future returns or subsume the risk-neutral tail risk measure in our tests. In this section, we address this question by considering several daily statistics constructed from the high-frequency SDF: its volatility, entropy, and Value-at-Risk. The first two measures capture different notions of variation in the SDF that may be informative about the price of risk, while the latter focuses explicitly on compensation for bad states.

For each of these SDF-based measures, we construct long–short portfolios by sorting stocks on their estimated exposures, obtained from daily regressions over the preceding 2 years. As reported in Table 10, sorting on these exposures yields statistically significant return spreads, indicating that variation in the estimated SDF is informative about risk premia "out-of-sample," that is, it helps explain return differences across a broad cross-section of stocks over the subsequent month. However, both the magnitude and statistical significance of these spreads are smaller than those obtained using the tail-risk-based portfolio in Table 6.

We then compute the alphas of the tail-risk-based portfolio relative to the portfolios formed on SDF-based measures and find that they remain statistically significant. This suggests that combining information from the high-frequency SDF with intra-day stock returns through the Hill estimator, as done to construct $\lambda_t^{\mathbb{Q}}$, produces a more robust and informative state variable for predicting future returns. This finding is consistent with our economic motivation: investors are particularly concerned about extreme downside risk and therefore demand higher compensation when both the level of tail risk and their aversion to such risk are elevated. Importantly, this evidence does not imply issues in the high-frequency SDF itself, which is designed to price the intra-day PCs returns on a given day, rather than to summarize all relevant information for longer horizons.

4.5 Robustness with respect to main hyperparameters

We now conduct a comprehensive battery of robustness tests to show that our results do not depend on specific modeling choices. In this subsection, we assess robustness with respect to the main hyperparameters involved in the construction of our tail measures: the percentile of the pooled intra-day return cross-section on day t to define the threshold u_t for the tail; the number J of PCs included as systematic factors in the estimation of the SDF; and the γ parameter determining the convexity of the SDF.

We start by reporting, in the Supplementary Appendix, complete tabulated results for alternative specifications that change one hyperparameter at a time. First, we estimate our tail risk measures considering alternative percentiles for u_t given by 0.5 or 5 percent, instead of the original first percentile. Focusing on the market return predictability, Supplementary Tables OA.7 and OA.8 report the results for 0.5 and 5 percent, respectively. As can be seen, considering a smaller (larger) percentile given by 0.5 percent (5 percent) slightly improves (decreases) the predictive power of both $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$. For both percentiles, the risk-neutral tail risk is still significant in all univariate regressions. A smaller (larger) percentile given by 0.5 percent (5 percent) also improves (deteriorates) the out-of-sample performance of both measures, as seen in Supplementary Tables OA.14 and OA.15. Importantly, the risk-neutral tail risk is more robust and significantly outperforms the benchmarks for most cases.

Table 10 Quintile portfolios sorted on exposures to measures based on the SDF.
Note: The table reports average returns and risk-adjusted alphas (in percentage points) for quintile portfolios sorted on exposures to various summary measures of the intra-day SDF. Panel A sorts stocks based on their most recent exposure to the standard deviation of the SDF (STD SDF). Panel B sorts on exposure to the Kullback–Leibler information criterion of the SDF (KLIC SDF), defined as $\mathbb{E}[m\log(m)]$. Panel C sorts on exposure to the 95% Value-at-Risk of the SDF (VaR SDF). Panel D reports alphas for portfolios sorted on exposure to the risk-neutral tail-risk measure $\lambda_t^{\mathbb{Q}}$, controlling for the STD, KLIC, and VaR SDF measures. The High–Low column reports the return differential between the top and bottom quintiles, with associated *t*-statistics computed using Newey–West robust standard errors. The sample spans January 1998 to December 2022.

	Low	2	3	4	High	High–Low	*t*-stat
Panel A: STD SDF							
Average Return	0.535	0.486	0.592	0.571	0.926	0.391	2.138
CAPM Alpha	−0.110	−0.073	0.027	−0.057	0.182	0.292	1.400
FF3 alpha	−0.174	−0.154	−0.060	−0.142	0.093	0.267	1.456
FF5 alpha	−0.071	−0.222	−0.151	−0.183	0.160	0.231	1.463
FF5 + Mom alpha	−0.062	−0.204	−0.126	−0.138	0.229	0.291	1.797
FF5 + Mom + Liq alpha	−0.051	−0.205	−0.142	−0.171	0.192	0.243	1.498
FF5 + Mom + Liq + LTV alpha	−0.053	−0.207	−0.142	−0.170	0.193	0.247	1.655
Panel B: KLIC SDF							
Average Return	0.513	0.510	0.550	0.601	0.926	0.413	2.260
CAPM Alpha	−0.157	−0.059	−0.014	−0.017	0.182	0.339	1.620
FF3 alpha	−0.220	−0.141	−0.098	−0.102	0.093	0.313	1.732
FF5 alpha	−0.096	−0.199	−0.195	−0.156	0.160	0.257	1.620
FF5 + Mom alpha	−0.087	−0.179	−0.166	−0.113	0.229	0.316	1.940
FF5 + Mom + Liq alpha	−0.075	−0.181	−0.180	−0.144	0.192	0.266	1.632
FF5 + Mom + Liq + LTV alpha	−0.077	−0.182	−0.180	−0.144	0.193	0.270	1.783
Panel C: VaR SDF							
Average Return	0.593	0.554	0.545	0.580	0.839	0.246	1.779
CAPM Alpha	−0.092	−0.039	−0.035	−0.008	0.143	0.235	1.468
FF3 alpha	−0.170	−0.127	−0.120	−0.086	0.066	0.236	1.524
FF5 alpha	−0.079	−0.199	−0.197	−0.130	0.140	0.219	1.358
FF5 + Mom alpha	−0.065	−0.166	−0.168	−0.094	0.192	0.257	1.563
FF5 + Mom + Liq alpha	−0.056	−0.167	−0.185	−0.116	0.147	0.203	1.217
FF5 + Mom + Liq + LTV alpha	−0.057	−0.167	−0.186	−0.117	0.147	0.204	1.216
Panel D: $\lambda_t^{\mathbb{Q}}$							
CAPM alpha + STD SDF	0.439	0.021	−0.076	−0.190	−0.327	−0.766	−5.079
FF3 alpha + STD SDF	0.364	−0.037	−0.128	−0.245	−0.393	−0.757	−5.241
FF5 alpha + STD SDF	0.367	−0.132	−0.219	−0.300	−0.220	−0.587	−4.941
CAPM alpha + KLIC SDF	0.432	0.010	−0.084	−0.194	−0.306	−0.738	−5.009
FF3 alpha + KLIC SDF	0.353	−0.045	−0.132	−0.246	−0.378	−0.731	−5.161
FF5 alpha + KLIC SDF	0.361	−0.136	−0.221	−0.300	−0.212	−0.573	−4.869
CAPM alpha + VaR SDF	0.479	0.061	−0.045	−0.168	−0.341	−0.821	−4.788
FF3 alpha + VaR SDF	0.382	−0.030	−0.128	−0.253	−0.407	−0.790	−4.994
FF5 alpha + VaR SDF	0.381	−0.124	−0.216	−0.304	−0.231	−0.612	−4.674

Supplementary Tables OA.21 and OA.22 report the corresponding results for predicting the variance risk premium. These are largely similar to the original results for $\lambda_t^{\mathbb{Q}}$. On the other hand, $\lambda_t^{\mathbb{P}}$ benefits from a smaller percentile, where it becomes significant in the joint regression with $\lambda_t^{\mathbb{Q}}$ at the 1-day and 1-week horizons, while it is still driven out at the 1-month horizon. Finally, Supplementary Tables OA.28 and OA.29 contain the portfolio sorts results. A smaller percentile benefits both measures: the risk-neutral tail factor yields even more significant alphas, while the physical tail factor is now able to produce significant spreads in returns. These spreads, however, are smaller than for $\lambda_t^{\mathbb{Q}}$ and still not monotonic across quintiles. Remarkably, even with the larger percentile, the risk-neutral tail factor retains monotonicity across quintiles and some statistical significance for the alphas.

Second, we assess robustness with respect to choices in the risk-neutralization procedure regarding the number of PCs and the γ parameter. We consider specifications using the top 5 or 10 PCs, instead of the top 7, and $\gamma = -2$, instead of $\gamma = -3$. Results across all our different analyses, reported in Supplementary Tables OA.9, OA.10, OA.11, OA.16, OA.17, OA.18, OA.23, OA.24, OA.25, OA.30, OA.31, OA.32, show that working with these alternative specifications has no meaningful impact on our results, which remain essentially the same.

Next, to provide a comprehensive assessment of how our main results for $\lambda_t^{\mathbb{Q}}$ vary with the hyperparameters, we examine the statistical significance of market and cross-sectional return predictability across all possible combinations of an extended range of choices, which includes percentiles 0.5, 1, and 5 for u_t, $J \in \{1,3,5,7,10\}$, and $\gamma \in \{-5,-4,-3,-2,-1,1\}$. When $J = 1$, the implied SDF is akin to a CAPM SDF since the first PC effectively captures the market factor, while $\gamma = 1$ yields a linear SDF (capped at zero to enforce no-arbitrage) with no preferences for higher moments of returns. Since $\lambda_t^{\mathbb{Q}}$ is always significant at the 1 percent level for predicting the *VRP* regardless of the specification, we relegate these results to Supplementary figure OA.1 and focus the discussion below on the equity premium and cross-sectional prediction exercises.

Figure 7 reports the t-statistics for the equity premium prediction. The weakest evidence is obtained when the tail threshold u_t is set to the 5th percentile. In this case, at the 1-day and 1-week horizons, statistical significance appears only for specifications with five PCs or more and highly negative γs, while at the 1-month horizon three PCs or more are enough for robust predictive power regardless of γ. In contrast, using smaller percentiles, given by 0.5 or 1, substantially strengthens the predictive power of $\lambda_t^{\mathbb{Q}}$. Across horizons, the 0.5 percentile yields strong significance for nearly all specifications, with one or three PCs performing as well as larger J. For the 1st percentile, the evidence is similar at the 1-week and 1-month horizons, while at the 1-day horizon five or seven PCs lead to more robust predictability across different γs. Overall, the equity premium results are robust across a wide range of specifications, but generally benefit from smaller percentiles, moderate numbers of PCs, and SDFs that embed aversion to downside risk.

A similar robustness pattern emerges in the cross-sectional analysis in figure 8. The most important hyperparameter is the percentile used to define u_t, followed by the number of PCs and, lastly, the value of γ. The weakest performance again arises for the 5th percentile, where t-statistics for the high-low portfolio average return and alphas hover around 2.5 and 2 (in absolute value), respectively, and drop below 2 (in absolute value) when only one PC is used. Using the 1st percentile already produces strong and generally significant results, even when only one PC is employed. The tightest threshold given by percentile 0.5 delivers the strongest cross-sectional evidence, with all specifications being highly significant. While the best results usually arise with five or seven PCs, the findings are broadly robust to using only one or three PCs. Taken together, the cross-sectional evidence reinforces the message from the equity premium analysis: predictive power is strongest when u_t targets more extreme left-

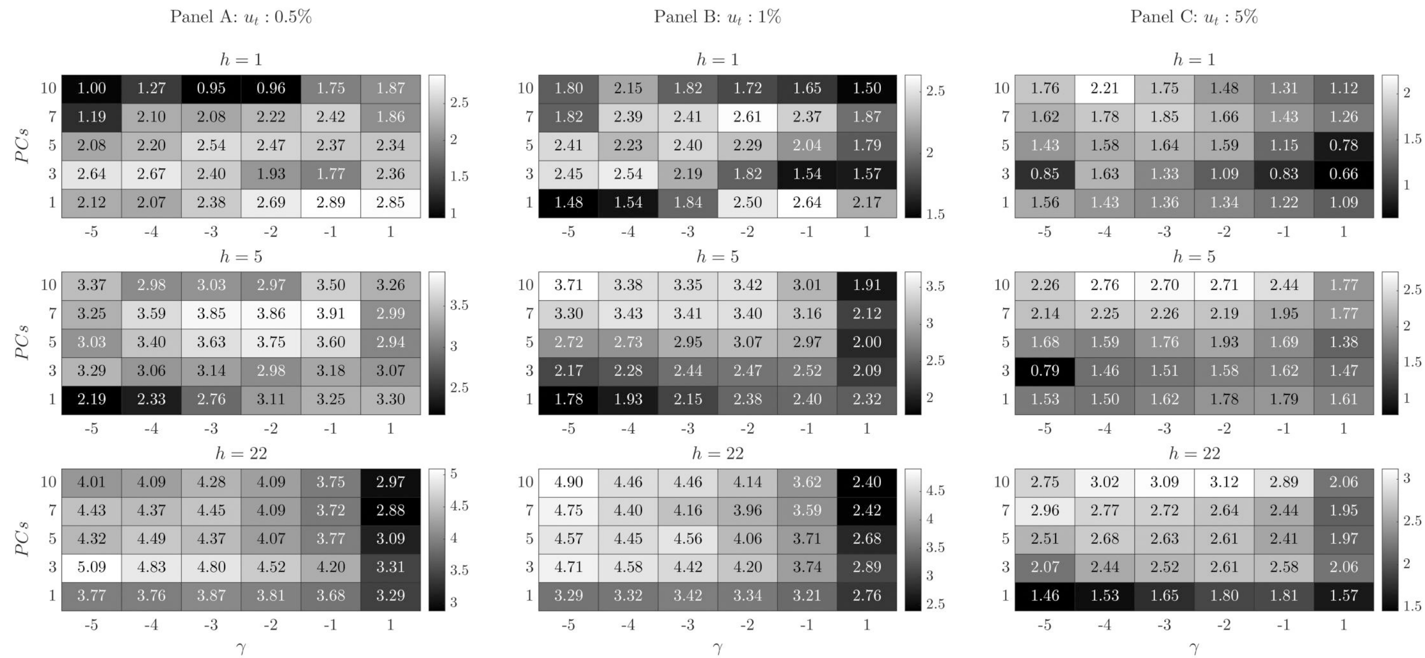

Figure 7 Robustness of risk-neutral tail risk to parameter choices: Equity premium prediction.

Note: The figure reports *t*-statistics from predictive regressions of the equity premium on $\lambda_t^{\mathbb{Q}}$. Each panel varies the threshold parameter (u_t) and horizon (h), and within each panel, rows correspond to the number of principal components used to estimate the SDF, while columns correspond to the γ values. The sample ranges from January 1996 to December 2022.

tail realizations, when an intermediate number of PCs is used, and when the SDF embeds downside risk aversion.

Finally, it is worth noting that even when using the simplest SDF specification given by one PC and $\gamma = 1$, which is essentially a linear CAPM SDF, the implied $\lambda_t^{\mathbb{Q}}$ still outperforms $\lambda_t^{\mathbb{P}}$. Specifically, the *t*-statistics for the equity premium prediction for $\lambda_t^{\mathbb{P}}$ of 1.76, 1.55, and 2.45 for the 1-day, 1-week, and 1-month horizons, are surpassed by the corresponding ones for $\lambda_t^{\mathbb{Q}}$ in figure 7 given by 2.17, 2.32, and 2.76. This is even more evident for the cross-sectional predictability, with a *t*-stat of -2.72 for the high-low portfolio average return based on the $J = 1/\gamma = 1$ $\lambda_t^{\mathbb{Q}}$ against -0.88 for $\lambda_t^{\mathbb{P}}$, reinforcing the importance of accounting for investors' preferences when computing tail risk.

Overall, the robustness tests above confirm that our main results are not driven by specific modeling choices and hold, both quantitatively and qualitatively, across several alternative specifications. More specifically, the risk-neutral tail risk is more robust than the physical tail risk to modifications in the methodology and essentially always contains important predictive information in the short-term about the equity premium, variance risk premium, and the cross-section of stock returns.

4.6 Additional robustness checks

In this subsection, we provide additional robustness checks for our analysis. First, we investigate what happens if we do not include the overnight return of each stock as an observation to compute the tail risk measures. Supplementary Tables OA.12 and OA.19 show that the in-sample and out-of-sample predictive power of both tail measures get slightly reduced without the overnight return, but the same conclusions as before remain: the risk-neutral tail risk is always significant and drives out the effect of the physical one in the joint predictive regressions. On the other hand, removing the overnight return improves the predictive power of both tail measures for the variance risk premium in Supplementary Table OA.26, where $\lambda_t^{\mathbb{P}}$ retains some significance in the joint regression with $\lambda_t^{\mathbb{Q}}$ for the 1-day and 1-week horizons. Regarding portfolio sorts, the evidence from Supplementary Table OA.33 is essentially the same as that from our main specification.

Second, we consider alternative versions of our tail risk measures based on returns that are standardized by their realized volatility adjusted for time-of-day effects, as in equation (A.1). Supplementary Table OA.13 reports the corresponding market return predictability results. For the 1-day horizon, the physical tail risk has stronger predictive power than the risk-neutral one. Even so, both are significant in the joint regression. For the 1-week and 1-month horizons, $\lambda_t^{\mathbb{Q}}$ becomes more significant, driving out the effect of $\lambda_t^{\mathbb{P}}$ in the joint 1-week ahead regression. In the out-of-sample prediction exercise in Supplementary Table OA.20, the tail measures have very similar performance, almost always beating the benchmarks. For predicting the variance risk premium in Supplementary Table OA.27, however, the results are substantially different for both measures. The risk-neutral tail risk continues to positively predict the *VRP*, with strong statistical significance. In contrast, the physical tail risk now has the wrong sign, predicting a smaller variance risk premium following an increase in tail risk. Finally, in terms of cross-sectional predictability in Supplementary Table OA.34, the predictive power of both measures gets reduced compared to our original results. However, the risk-neutral tail risk still retains significance at the 10 percent level for alphas with respect to most models and for monotonicity across quintiles according to the MR Down test.

Third, we consider value-weighted instead of equally-weighted portfolios in our cross-sectional predictability analysis with $\lambda_t^{\mathbb{Q}}$. Supplementary figure OA.2 reports the *t*-statistics for the average return and alphas of the high-minus-low value-weighted portfolio based on

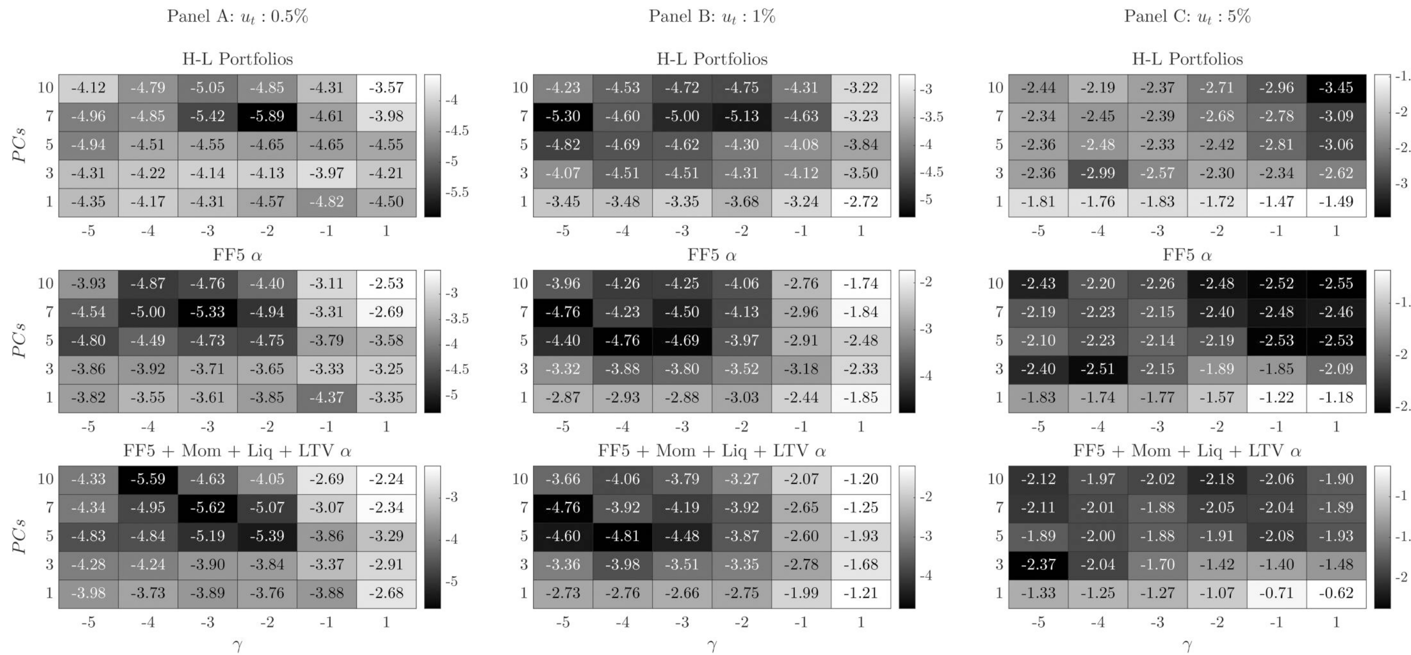

Figure 8 Robustness of risk-neutral tail risk to parameter choices: Cross-sectional predictability.

Note: The figure reports *t*-statistics for the average return or alpha of the high-minus-low portfolio based on sorting stocks on their recent exposure to $\lambda_t^{\mathbb{Q}}$. Each panel varies the threshold parameter (u_t) and alpha, and within each panel, rows correspond to the number of principal components used to estimate the SDF, while columns correspond to the γ values. The sample ranges from January 1998 to December 2022.

$\lambda_t^{\mathbb{Q}}$ across different specifications and percentiles 0.5 and 1, which we have previously shown to be preferable. Results are mostly robust to value-weighting stocks within portfolios. For percentile 1, robust predictability is concentrated in specifications with three to four PCs, although alphas with respect to the most complete factor model are usually insignificant. For percentile 0.5, average returns and alphas are almost always significant, with the exception of the specification with ten PCs, which produces weaker results.

Fourth, to highlight the importance of using high-frequency stock returns to compute the Hill estimator at a daily frequency, we compare our baseline $\lambda_t^{\mathbb{P}}$ with an alternative measure constructed using only daily stock returns, that is, one observation per stock per day. Supplementary figure OA.3 illustrates the time series dynamics of the two measures and shows that relying on daily returns produces a tail risk measure that is substantially noisier and more volatile. Supplementary Table OA.38 further reports the results from predictive regressions of the equity premium using both measures. As can be seen, the tail measure based on daily returns delivers coefficients with the wrong sign, lacks statistical significance, and yields markedly lower R^2 values.

Fifth, we examine the sensitivity of our results to the filter imposed on zero returns. In our baseline specification, we exclude stocks that, on a given day, exhibit fewer than thirty-five nonzero intra-day return observations. As a robustness check, we alternatively apply more lenient and more stringent filters, retaining stocks with at least twenty or at least fifty nonzero observations, respectively. Supplementary Tables OA.39 and OA.40 report the corresponding results for equity premium predictability using $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$. Overall, the findings are both qualitatively and quantitatively very similar to those in the baseline specification in Table 2, indicating that our results are not sensitive to the choice of the zero-return filter.

Sixth and finally, to shed further light on the properties of the endogenous portfolio $\theta_\gamma^* F_{n,t}$ that enters the estimated SDF, Supplementary figure OA.4 reports the daily Sharpe ratios of the portfolio constructed each day, smoothed over time using a moving average, for different numbers of factors. Focusing on the top panels, the Sharpe ratios obtained with a single factor closely track those of the SPY, consistent with the fact that the first PC effectively captures the market factor. When additional factors are included, Sharpe ratios are generally high, as would be expected in an in-sample setting. The bottom panels report out-of-sample Sharpe ratios, computed by holding the PCs and portfolio weights fixed and applying them to returns on the following day. These out-of-sample Sharpe ratios, which provide a more appropriate metric for what investors can achieve in practice, are of economically reasonable magnitudes. Moreover, Sharpe ratios tend to decline during crisis periods, consistent with reduced risk-adjusted performance during market stress.

5. Conclusion

We introduce a novel left-tail risk measure at a daily frequency by combining high-frequency returns of a large cross-section of stocks with a risk-neutralization algorithm. We use our measure to shed new light on the effects of tail risk on asset prices in the short-term, with a particular focus on disentangling to what extent these effects depend on information coming from the physical measure, under which asset prices are observed, and the risk-neutral measure, which incorporates investors' preferences.

We find that the economic valuation of tail risk is an important determinant of the equity premium and the variance risk premium at short horizons, where both are high following increases in tail risk. In addition, short-term exposure to tail risk is priced in the cross-section of stocks. A tradable tail factor built by sorting stocks on their recent exposure to tail risk produces significant spreads in stock returns that cannot be explained by standard factor models. Moreover, exposures to our tail risk measure are strongly significant in Fama–MacBeth regressions. Incorporating investors' preferences in the estimation of tail risk is fundamental to our findings: the predictability afforded by the physical tail risk is weaker and generally subsumed by its risk-neutral counterpart.

While we use our methodology to measure aggregate left-tail risk in the U.S. stock market, it can be readily applied in a number of different contexts. For instance, our approach can be similarly employed to estimate the economic perception of right tail risk. This can potentially be useful to explain cross-sectional anomalies that are exposed to the right tail or related to lottery preferences, such as idiosyncratic volatility. Furthermore, given that our method does not rely on information from option prices, it can be applied to other asset classes for which option markets are relatively illiquid or inexistent, such as cryptocurrencies or corporate bonds. Relatedly, our procedure could be especially helpful in extracting tail risk for particular portfolios or indices of stocks for which basket options are not available. We leave these interesting venues for future research.

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Supplementary material

Supplementary material is available at *Review of Finance* online.

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Conflict of interest

None declared.

Data availability

The data underlying this article were obtained under license from various data providers as described in the article. As the data are proprietary and cannot be shared publicly, pseudo-data illustrating the required input format, along with the main codes, are shared with the *Review of Finance*.

References

Aït-Sahalia, Y., I. Kalnina, and D. Xiu. 2020. "High-Frequency Factor Models and Regressions." *Journal of Econometrics* 216: 86–105.

Aït-Sahalia, Y., and A. W. Lo. 2000. "Nonparametric Risk Management and Implied Risk Aversion." *Journal of Econometrics* 94: 9–51.

Aït-Sahalia, Y., and D. Xiu. 2017. "Using Principal Component Analysis to Estimate a High Dimensional Factor Model with High-Frequency Data." *Journal of Econometrics* 201: 384–99.

Aït-Sahalia, Y., and D. Xiu. 2019. "A Hausman Test for the Presence of Market Microstructure Noise in High Frequency Data." *Journal of Econometrics* 211: 176–205.

Almeida, C., K. Ardison, G. Freire, R. Garcia, and P. Orłowski. 2024. "High-Frequency Tail Risk Premium and Stock Return Predictability." *Journal of Financial and Quantitative Analysis* 59: 3633–70.

Almeida, C., K. Ardison, R. Garcia, and J. Vicente. 2017. "Nonparametric Tail Risk, Stock Returns, and the Macroeconomy." *Journal of Financial Econometrics* 15: 333–76.

Almeida, C., and G. Freire. 2022. "Pricing of Index Options in Incomplete Markets." *Journal of Financial Economics* 144: 174–205.

Almeida, C., G. Freire, and R. Hizmeri. (2024). "0DTE asset pricing." *Working Paper*.

Almeida, C., and R. Garcia. 2017. "Economic Implications of Nonlinear Pricing Kernels." *Management Science* 63: 3361–80.

Amaya, D., P. Christoffersen, K. Jacobs, and A. Vasquez. 2015. "Does Realized Skewness Predict the Cross-Section of Equity Returns?" *Journal of Financial Economics* 118: 135–67.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens. 2001. "The Distribution of Realized Stock Return Volatility." *Journal of Financial Economics* 61: 43–76.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys. 2003. "Modeling and Forecasting Realized Volatility." *Econometrica* 71: 579–625.

Andersen, T. G., Y. Ding, and V. Todorov. (2024). "The Granular Origins of Tail Dispersion Risk." *Working Paper*.

Andersen, T. G., N. Fusari, and V. Todorov. 2015. "The Risk Premia Embedded in Index Options." *Journal of Financial Economics* 117: 558–84.

Andersen, T. G., N. Fusari, and V. Todorov. 2017. "Short-Term Market Risks Implied by Weekly Options." *Journal of Finance* 72: 1335–86.

Andersen, T. G., N. Fusari, and V. Todorov. 2020. "The Pricing of Tail Risk and the Equity Premium: Evidence from International Option Markets." *Journal of Business & Economic Statistics* 38: 662–78.

Andersen, T. G., V. Todorov, and M. Ubukata. 2021. "Tail Risk and Return Predictability for the Japanese Equity Market." *Journal of Econometrics* 222: 344–63.

Ang, A., J. Chen, and Y. Xing. 2006. "Downside Risk." *Review of Financial Studies* 19: 1191–239.

Atilgan, Y., T. G. Bali, K. O. Demirtas, and A. D. Gunaydin. 2020. "Left-Tail Momentum: Underreaction to Bad News, Costly Arbitrage and Equity Returns." *Journal of Financial Economics* 135: 725–53.

Bali, T. G., N. Cakici, and R. F. Whitelaw. 2011. "Maxing Out: Stocks as Lotteries and the Cross-Section of Expected Returns." *Journal of Financial Economics* 99: 427–46.

Bali, T. G., N. Cakici, and R. F. Whitelaw. 2014. "Hybrid Tail Risk and Expected Stock Returns: When Does the Tail Wag the Dog?" *Review of Asset Pricing Studies* 4: 206–46.
Bali, T. G., K. O. Demirtas, and H. Levy. 2009. "Is There an Intertemporal Relation between Downside Risk and Expected Returns?" *Journal of Financial and Quantitative Analysis* 44: 883–909.
Bandi, F. M., N. Fusari, and R. Renò. (2023). "0DTE Option Pricing." *Working Paper*.
Bandi, F. M., A. Kolokolov, D. Pirino, and R. Renò. 2020. "Zeros." *Management Science* 66: 3466–79.
Bandi, F. M., D. Pirino, and R. Renò. 2017. "Excess Idle Time." *Econometrica* 85: 1793–846.
Barndorff-Nielsen, O. E., and N. Shephard. 2004. "Power and Bipower Variation with Stochastic Volatility and Jumps." *Journal of Financial Econometrics* 2: 1–37.
Barro, R. J. 2006. "Rare Disasters and Asset Markets in the Twentieth Century." *Quarterly Journal of Economics* 121: 823–66.
Bekaert, G., and M. Hoerova. 2014. "The VIX, the Variance Premium and Stock Market Volatility." *Journal of Econometrics* 183: 181–92.
Bekaert, G., M. Hoerova, and M. L. Duca. 2013. "Risk, Uncertainty and Monetary Policy." *Journal of Monetary Economics* 60: 771–88.
Bégin, J.-F., C. Dorion, and G. Gauthier. 2020. "Idiosyncratic Jump Risk Matters: Evidence from Equity Returns and Options." *The Review of Financial Studies* 33: 155–211.
Bliss, R. R., and N. Panigirtzoglou. 2004. "Option-Implied Risk Aversion Estimates." *Journal of Finance* 59: 407–46.
Bollerslev, T., S. Z. Li, and V. Todorov. 2016. "Roughing Up Beta: Continuous versus Discontinuous Betas and the Cross Section of Expected Stock Returns." *Journal of Financial Economics* 120: 464–90.
Bollerslev, T., A. J. Patton, and R. Quaedvlieg. 2022. "Realized Semibetas: Disentangling "Good" and "Bad" Downside Risks." *Journal of Financial Economics* 144: 227–46.
Bollerslev, T., G. Tauchen, and H. Zhou. 2009. "Expected Stock Returns and Variance Risk Premia." *Review of Financial Studies* 22: 4463–92.
Bollerslev, T., and V. Todorov. 2011. "Tails, Fears, and Risk Premia." *Journal of Finance* 66:2165–211.
Bollerslev, T., V. Todorov, and L. Xu. 2015. "Tail Risk Premia and Return Predictability." *Journal of Financial Economics* 118: 113–34.
Campbell, J. Y., and J. H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107: 205–51.
Campbell, J. Y., and S. B. Thompson. 2008. "Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?" *Review of Financial Studies* 21: 1509–31.
Carhart, M. M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance* 52: 57–82.
Chabi-Yo, F., S. Ruenzi, and F. Weigert. 2018. "Crash Sensitivity and the Cross Section of Expected Stock Returns." *Journal of Financial and Quantitative Analysis* 53: 1059–100.
Clark, T. E., and K. D. West. 2007. "Approximately Normal Tests for Equal Predictive Accuracy in Nested Models." *Journal of Econometrics* 138: 291–311.
Corsi, F. 2008. "A Simple Approximate Long-Memory Model of Realized Volatility." *Journal of Financial Econometrics* 7: 174–96.
Cressie, N., and T. R. Read. 1984. "Multinomial Goodness-of-Fit Tests." *Journal of the Royal Statistical Society: Series B (Methodological)* 46: 440–64.
Danielsson, J. 2011. *Financial Risk Forecasting: The Theory and Practice of Forecasting Market Risk with Implementation in R and Matlab*. Chichester, UK: John Wiley & Sons.
Fama, E. F., and K. R. French. 2015. "A Five-Factor Asset Pricing Model." *Journal of Financial Economics* 116: 1–22.

Fama, E. F., and J. D. MacBeth. 1973. “Risk, Return, and Equilibrium: empirical Tests.” *Journal of Political Economy* 81: 607–36.

Freire, G. 2021. “Tail Risk and Investors’ Concerns: Evidence from Brazil.” *North American Journal of Economics and Finance* 58: 101519.

Gabaix, X. 2012. “Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance.” *Quarterly Journal of Economics* 127: 645–700.

Gormsen, N. J., and C. S. Jensen. 2024. “Higher-Moment Risk.” *Journal of Finance*, forthcoming.

Graham, J. R., and C. Harvey. 2001. “The Theory and Practice of Corporate Finance: Evidence from the Field.” *Journal of Financial Economics* 60:187–243.

Hansen, L. P., and R. Jagannathan. 1991. “Implications of Security Market Data for Models of Dynamic Economies.” *Journal of Political Economy* 99: 225–62.

Hill, B. M. 1975. “A Simple General Approach to Inference about the Tail of a Distribution.” *Annals of Statistics*, 3: 1174.

Huang, D., and G. Zhou. 2017. “Upper Bounds on Return Predictability.” *Journal of Financial and Quantitative Analysis* 52: 401–25.

Jegadeesh, N. 1990. “Evidence of Predictable Behavior of Security Returns.” *Journal of Finance* 45: 881–98.

Jegadeesh, N., and S. Titman. 1993. “Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency.” *Journal of Finance* 48: 65–91.

Kelly, B., and H. Jiang. 2014. “Tail Risk and Asset Prices.” *Review of Financial Studies* 27: 2841–71.

Kimball, M. S. 1990. “Precautionary Saving in the Small and in the Large.” *Econometrica* 58: 53–73.

Kozak, S., S. Nagel, and S. Santosh. 2018. “Interpreting Factor Models.” *Journal of Finance* 73: 1183–223.

Kozak, S., S. Nagel, and S. Santosh. 2020. “Shrinking the Cross-Section.” *Journal of Financial Economics* 135: 271–92.

Lehmann, B. N. 1990. “Fads, Martingales, and Market Efficiency.” *Quarterly Journal of Economics* 105: 1–28.

Martin, I. 2017. “What is the Expected Return on the Market?” *Quarterly Journal of Economics* 132: 367–433.

Pástor, L., and R. F. Stambaugh. 2003. “Liquidity Risk and Expected Stock Returns.” *Journal of Political Economy* 111: 642–85.

Patton, A. J., and A. Timmermann. 2010. “Monotonicity in Asset Returns: New Tests with Applications to the Term Structure, the CAPM, and Portfolio Sorts.” *Journal of Financial Economics* 98: 605–25.

Pelger, M. 2020. “Understanding Systematic Risk: A High-Frequency Approach.” *Journal of Finance* 75: 2179–220.

Pettenuzzo, D., A. Timmermann, and R. Valkanov. 2014. “Forecasting Stock Returns under Economic Constraints.” *Journal of Financial Economics* 114: 517–53.

Ross, S. A. 2005. *Neoclassical Finance*. Princeton, NJ: Princeton University Press.

Schneider, P., and F. Trojani. 2015. “Fear Trading.” *Swiss Finance Institute Research Paper* (15-03).

Stutzer, M. 1995. “A Bayesian Approach to Diagnosis of Asset Pricing Models.” *Journal of Econometrics* 68: 367–97.

Wachter, J. 2013. “Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?” *Journal of Finance* 68: 987–1035.

Weller, B. M. 2019. “Measuring Tail Risks at High Frequency.” *Review of Financial Studies* 32: 3571–616.

Appendix A: standardized tail risk measures

As can be seen in equations (2) and (4), the standard Hill estimator relies on a threshold u_t that is the same across stocks and constant over the day. This is such that stocks that are more volatile on day t might dominate the tail risk measures, as they will be more likely to experience large negative return realizations falling below the threshold than low-volatility stocks. Moreover, given that volatility tends to be higher at the market open and close, these times of the day might be overrepresented in the tail measures as returns observed during those times will be more likely to fall below u_t. To control for such cross-sectional and time-of-day patterns in volatility, we consider standardized versions of the physical and risk-neutral Hill estimators.

More specifically, for each stock, we standardize its returns (either raw or risk-neutralized) on day t using a measure of the integrated volatility adjusted by the time-of-day factor:

$$\widehat{R}_{t,n}i = \frac{R^i_{t,i}}{\sqrt{N^{-1}(BV^i_t \quad RV^i_t) \cdot TOD^i_n}}, \tag{A.1}$$

where the Realized Variance (RV) and Bipower variation (BV) are computed as defined in Appendix B.[29]

Using these realized measures, we follow Bollerslev and Todorov (2011) to estimate the Time-of-Day (TOD) volatility pattern for each stock as:

$$TOD^i_n = \frac{N\sum_{t=1}^{T}|R^i_{n,t}|^2\mathbb{I}(|R^i_{n,t}| \leq \alpha\sqrt{BV^i_t \quad RV^i_t}N^{-\varpi})}{\sum_{t=1}^{T}\sum_{n=1}^{N}|R^i_{n,t}|^2\mathbb{I}(|R^i_{n,t}| \leq \alpha\sqrt{BV^i_t \quad RV^i_t}N^{-\varpi})}, \tag{A.2}$$

where $\alpha = 3$ and $\varpi = 0.49$.[30] Then, we compute the Hill estimator using the standardized returns for all stocks, but now with a constant threshold u as the returns are already in standard deviation units. We set $u = -3.5$, that is, the tail is defined as the stock returns falling below -3.5 standard deviations of their respective day-t distributions.

Appendix B: Variables definitions

- Reversal (REV): following Jegadeesh (1990) and Lehmann (1990), the short-term reversal variable is defined as the weekly market return over the previous week from day $t-4$ to day t.
- Momentum (MOM): following Jegadeesh and Titman (1993), the momentum variable at the end of day t is defined as the compound gross market return from day $t-252$ through day $t-21$, skipping the short-term reversal month.
- Variance Risk Premium (VRP): we compute the variance risk premium as a short position in a variance swap, namely, as the difference between risk-neutral and physical

[29] We consider the minimum ( ) between the RV_t and BV_t as a finite-sample adjustment, since the asymptotic limit of BV^i_t is always below that of RV^i_t.

[30] Figure A1 depicts the average time-of-day factor across all stocks in our sample, illustrating the well-documented U-shaped pattern in the average intra-day volatility.

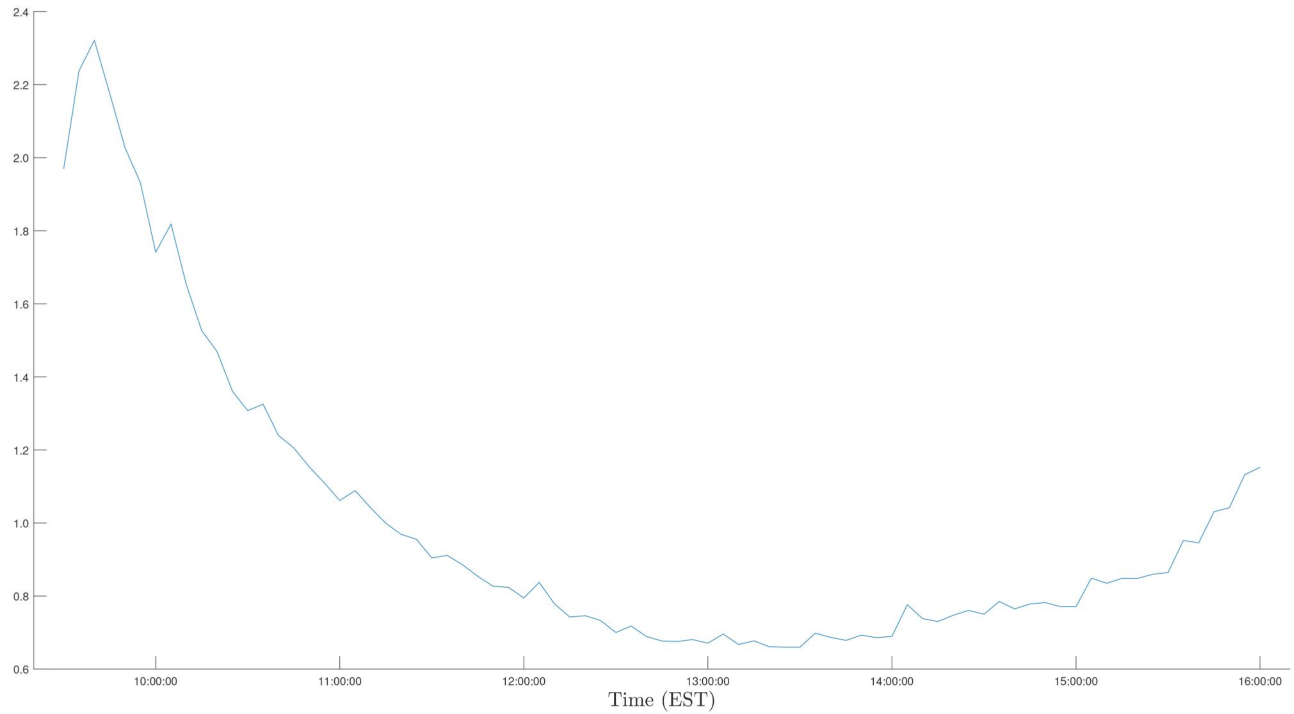

Figure A1 Time-of-day factor.

Note: The figure depicts the average time-of-day factor for all the stocks in our sample, estimated based on 5-min high-frequency returns from January 1996 to December 2022.

expectations of the variance of market returns (e.g., Bollerslev, Tauchen, and Zhou 2009; Bekaert and Hoerova 2014):

$$VRP_t = VIX_t^2 - \mathbb{E}_t[RV_{t+1}],$$

where VIX_t is the CBOE volatility index, and RV_{t+1} is the out-of-sample forecast of the SPX realized variance measured over the next month. We obtain this measure from Marie Hoerova's website.

- Maximum daily return (MAX): the MAX variable is defined as the largest total daily market return observed over the previous week (see Bali, Cakici, and Whitelaw 2011).
- Minimum daily return (MIN): the MIN variable is defined as the smallest total daily market return observed over the previous week (see Bali, Cakici, and Whitelaw 2011).
- Realized Variance (RV): the realized variance is defined as the sum of the intra-day squared returns (e.g., Andersen et al. 2001, 2003):

$$RV_t = \sum_{n=1}^{N} R_{t,n}^2,$$

where $R_{t,n}$ denotes the log-return on the S&P 500 index over the n-th intra-daily time interval on day t.

- Realized Skewness (RSK): the RSK is the ex post daily realized skewness based on intra-day market returns standardized by the realized variance (e.g., Amaya et al. 2015):

$$RSK_t = \frac{\sqrt{N}\sum_{n=1}^{N} R_{t,n}^3}{RV_t^{3/2}}.$$

- Realized Kurtosis (RKU): the RKU is the ex post daily realized kurtosis based on intra-day market returns standardized by the variance (e.g., Amaya et al. 2015):

$$RKU_t = \frac{N\sum_{n=1}^{N} R_{t,n}^4}{RV_t^2}.$$

- Jump Variation (JV): the jump variation is defined as the difference between the RV and a consistent measure of the integrated variance, such as the bipower variation (BV) of Barndorff-Nielsen and Shephard (2004):

$$JV_t = \max(RV_t - BV_t, 0),$$

where $BV_t = \pi/2(N/(N-1))\sum_{n=2}^{N} |R_{t,n}||R_{t,n-1}|$.

- Left-Tail Volatility (LTV): the left tail volatility proposed by Bollerslev, Todorov, and Xu (2015) is an option-implied measure of short-horizon downside tail risk obtained from short-dated out-of-the-money put options.
- Right-Tail Volatility (RTV): the right tail volatility proposed by Bollerslev, Todorov, and Xu (2015) is an option-implied measure of short-horizon upside tail risk obtained from short-dated out-of-the-money call options.

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